Pathways Beyond the Standard Model

Chapter 9 Pathways Beyond the Standard Model

1 Introduction

Over the past 30 years or so, high energy physics experiments have systematically explored the behavior of the strong, electromagnetic and weak interactions. For the strong interactions, QCD is generally accepted as the correct description, and research on QCD has shifted to its application to special regimes such as diffractive and exclusive processes and the quark-gluon plasma. For the electromagnetic and weak interactions, the progress of the past decade on W, Z, top, and neutrino physics has demonstrated that their structure is understood with high precision.

Our current picture of the electroweak interactions requires spontaneous gauge symmetry breaking. As yet, there is no direct evidence on the means by which the gauge symmetry is broken. It is remarkable that all of the evidence accumulated to date is consistent with the Standard Model (SM) in which this symmetry breaking is due to a single elementary scalar field, the Higgs field, which generates the masses of the W and Z bosons and the quarks and leptons.

However, many features of this simple theory are inadequate. The Higgs field is an *ad hoc* addition to the SM. Its mass and symmetry-breaking expectation value are put in by hand. The quark and lepton masses are generated by arbitrary couplings to the Higgs field. The existence of three generations of quarks and leptons is not explained, nor is the dramatic lack of symmetry in the masses and mixings of these generations.

To explain these features, it is necessary to extend the SM. These extensions, in turn, predict new particles and phenomena. The compelling motivation for new experiments at the highest energies is to discover these phenomena and then to decipher them, so that we can learn the nature of the new laws of physics with which they are associated.

In this document, we are exploring the physics case for a next-generation e^+e^- linear collider. To make this case, it is necessary to demonstrate that the linear collider can have an important impact on our understanding of these new phenomena. The argument should be made broadly for models of new physics covering the whole range of possibilities allowed from our current knowledge. It should take into account new information that we will learn from the Tevatron and LHC experiments which will be done before the linear collider is completed.

Our purpose in this chapter is to give an overview of possibilities for new physics beyond the SM. Our emphasis will be on general orientation to the pathways that one might follow. We will then explain the relevance of the linear collider measurements to each possible scenario. We encourage the reader to consult the relevant chapter of the 'Sourcebook', Chapters 3–8, to see how each quantity we discuss is measured at a linear collider and why the experimental precision that we expect is justified.

The essay is organized as follows: In Section 2, we discuss the general principle that we use to organize models of new physics. In Sections 3 and 4, we discuss models of new physics in the typical dichotomy used since the 1980's: on the one hand, models with supersymmetry, on the other hand, models with new strong interactions at the TeV scale. In Section 5, we discuss a new class of models for which the key ingredient is the existence of extra spatial dimensions. It is now understood that these models stand on the same footing as the more traditional schemes and, in fact, address certain of their weaknesses. Section 6 gives some conclusions.

2 Beyond the Standard Model

We first discuss some general principles regarding physics beyond the Standard Model.

From an experimental point of view, it is necessary to study the interactions of the observed particles at higher energies and with higher accuracy. This may lead to the discovery of new particles, in which case we need to study their spectrum and determine their interactions. Alternatively, it may lead to the observation of anomalous properties of the observed particles, in which case we could infer the existence of new particles or phenomena responsible for these effects. After this information is obtained in experiments, we must attempt to reconstruct the structure of the underlying theory. The linear collider is a crucial complement to the LHC in ensuring that the experimental information is extensive and precise enough for this goal to be achieved.

From the theoretical point of view, different ideas lead to models that provide challenges to this experimental program. To discuss the range of possible models, an organizing principle is needed. We will organize our discussion around the major question that we believe most strongly motivates new physics at the TeV scale. This is the *stability crisis* in the SM explanation for electroweak symmetry breaking. In technical terms, this is the problem that the Higgs boson mass is extremely sensitive to physics at very high energy scales. In the SM, the effect of quantum fields at the energy scale M is an additive contribution to the Higgs boson mass term of order M^2. More physically, this is the problem that not only the magnitude but even the sign of the Higgs boson mass term is not predicted in the SM, so that the SM cannot explain *why* the electroweak gauge symmetry is broken. From either perspective, this problem suggests that the SM is a dramatically incomplete picture of electroweak symmetry breaking. It is for this reason that we believe that new physics must appear at the TeV scale. We expect that the physics will be more exciting than simply the production of some random new particles. The solution of the stability crisis will involve completely new principles of physics. These principles will be reflected in the spectrum and properties of the new particles, and in their interactions. Much as the discovery of the J/ψ convincingly brought together many different elements of the SM in a coherent picture, so the discovery and study of these new states will spur us on to the construction of a new theory that will displace the SM.

We will use the idea of solving the stability crisis to guide our classification of the various models of new physics. The three approaches to this problem that have received the most study are supersymmetry, strongly coupled theories, and extra dimensions. The common theme in all three proposed solutions is that additional particle states and dynamics must be present near the electroweak scale. We briefly describe each approach, summarizing in each case the types of new interactions expected and the key experimental issues they raise.

Each possible model of new physics must be approached from the viewpoint expressed at the beginning of this section, that of dissecting experimentally the spectrum of new particles and their interactions. We take particular note of the important strengths that the linear collider brings to disentangling the physics of these models. We will see that, in most cases, the linear collider not only contributes but is *essential* to forming this experimental picture. Even if none of the specific models we discuss here is actually realized in Nature, this exercise illustrates the importance of the linear collider in unraveling the new world beyond the SM.

3 Supersymmetry

One attempt to cure the stability crisis of the Higgs field is to introduce a new symmetry—supersymmetry—which relates fermions and bosons. To realize this symmetry in Nature, there must exist supersymmetry partners for each of the known SM particles. Further, supersymmetry must be broken in the ground state so that these superpartners are more massive than ordinary particles. The Higgs mass terms are then not sensitive to mass scales above the superpartner masses. The Higgs field vacuum expectation value is naturally of order 100 GeV if the superpartner masses are also near this energy scale.

The existence of superpartners implies a rich program for future accelerators. The phenomenology of supersymmetry has been studied in great detail in the literature. Dozens of papers have been written on the technical ability of linear collider experiments to discover and study supersymmetric theories of many different forms. This material is reviewed systematically in Chapter 4 of this book. Different patterns of supersymmetry breaking masses can yield substantially different phenomenology at a high-energy collider. Supersymmetry is not a dot on the theoretical landscape, but rather contains a tremendously varied range of possibilities to be searched for and studied at all available high-energy collider facilities.

In the remainder of this section, we summarize the most important issues for the study of supersymmetry and the relevant measurements that can be done at a linear collider. It is important to keep in mind that we are likely to be surprised with the spectrum that Nature ultimately gives us. The linear collider's ability to cleanly disentangle the superpartner mass spectrum and couplings would be extremely important when the surprises occur. Of course, this is relevant only if the linear collider has sufficiently high center-of-mass energy to produce the superpartners. Section 2 of Chapter 4 reviews the expectations for the masses of superpartners and gives estimates of what center-of-mass energies should be required.

Mass measurements of accessible sparticles. If supersymmetry is relevant for electroweak symmetry breaking, then some of the superpartners should be discovered at the LHC. Furthermore, the experiments at the LHC should be able to accurately measure some masses or mass differences of the SUSY spectrum. This issue is reviewed in Chapter 4, Section 7. However, the systematic measurement of the SUSY spectrum requires a linear collider.

Superpartner masses are measured at a linear collider in three main ways: from distributions of the products of an on-shell superpartner decay, from threshold scans, and from contributions of virtual superpartners to cross sections or decay amplitudes. When sleptons, charginos, and neutralinos are produced on-shell, their masses will typically be measured to within about 1%. Even if the lightest neutralino LSP is not directly observed, its mass should be measurable to within 1% from these kinematic distributions. Threshold scans of sleptons in e^+e^- collisions and especially in e^-e^- collisions may yield mass measurements to within one part in a thousand. Indirect off-shell mass measurements are more model-dependent but have power in specific applications. For example, the t-channel sneutrino contribution to chargino pair production may allow the presence of the sneutrino to be deduced when its mass is as high as twice the center-of-mass energy of the collider. These techniques are reviewed in more detail in Chapter 4, Section 3.

Slepton and squark quantum numbers and mixing angles. When sparticle mixing can be ignored, the cross sections for pair production of squarks and sleptons at a linear collider are precisely determined by the SM quantum numbers. This should allow unambiguous checks of the quantum numbers and spins for sparticles of the first two generations. In particular, it is straightforward to distinguish the superpartners of left- and right-handed species (*e.g.*, $\tilde{e}_L$ from $\tilde{e}_R$) by cross section measurements with polarized beams. Third-generation sleptons and squarks are likely to be the most strongly mixed scalars of supersymmetry, forming mass eigenstates $\tilde{\tau}_{1,2}$, $\tilde{b}_{1,2}$, and $\tilde{t}_{1,2}$. Separation of these eigenstates and accurate measurement of their masses are difficult at the Tevatron and LHC but present no extraordinary problems to a linear collider. By combining direct mass measurements with polarization asymmetries for the production of these sparticles, we can determine the mixing angle needed to form the observed mass eigenstates from the left- and right-handed weak-interaction eigenstates. The uncertainty in this determination depends on the parameters of the theory, but it has been demonstrated for some cases that the error is lower than 1%.

Chargino/neutralino parameters. The neutralino and chargino states may be strongly mixed combinations of gauge boson and Higgs boson superpartners. The mass matrix is determined by four parameters of the underlying Lagrangian: M_1 (bino mass), M_2 (wino mass), μ (supersymmetric higgsino mass) and $\tan\beta$ (ratio of Higgs vacuum expectation values). Precision measurements of masses, mixing angles, and couplings associated with chargino and neutralino production can supply the information to determine these four important underlying parameters of supersymmetry. For example, measurements of chargino production alone can, in some cases, determine $\tan\beta$ to better than 10% with only 100 fb^{-1} of data. The parameters M_1, M_2, and possibly μ can be determined at the percent level in large portions of the accessible supersymmetry parameter space.

Coupling relations. To establish supersymmetry as a principle of Nature, it is important to verify some of the symmetry relations that that principle predicts. An essential consequence of supersymmetry is that the couplings of sparticles to gauginos are equal to the corresponding couplings of particles to gauge bosons. It has been demonstrated that this equality can be tested at a linear collider to levels better than 1% for weakly interacting sparticles. The precision is sufficiently good that one can even contemplate measuring the tiny deviations from coupling equivalence that are caused by supersymmetry-breaking effects in loop corrections. This can give an estimate of the masses of unobserved sparticles with mass well above the collider energy, in the same way that the current precision measurements predict the mass of the Higgs. This issue is reviewed in Chapter 4, Section 4.

CP violating phases. The SM apparently does not have enough CP violation to account for the baryon asymmetry in the universe. Supersymmetry has parameters that may introduce additional sources of CP violation into the theory. Testing for the existence of such phases would be an important part of a full supersymmetry program. It has been shown that the linear collider can determine evidence for additional non-zero CP-violating phases in supersymmetric theories if the phases are large enough ($\phi_i \sim 0.1$), even accounting for the constraints from electric dipole moment measurements.

Lepton number violation. Recent data suggest that neutrinos have non-zero masses and mixings. This implies that non-zero lepton flavor angles should be present for leptons, in parallel with the CKM angles for the quarks. These rotation angles are difficult to measure using high-energy leptons because neutrinos are invisible and are summed over in most observables. However, these angles could be detected from superpartner decays, such as $\tilde{\mu}^+\tilde{\mu}^- \to e^+\mu^-\tilde{\chi}_1^0\tilde{\chi}_1^0$. A linear collider can use these measurements to probe the lepton flavor angles with greater sensitivity than any existing experiment in some parts of parameter space.

Complete spectrum. The LHC will be a wonderful machine for the discovery of many supersymmetric sparticles in large regions of parameter space. The linear collider can add to the superpartner discoveries at the LHC by detecting states that are not straightforward to observe in the *pp* environment. The discovery abilities of the linear collider begin to be important at energies above LEPII and become increasingly important at energies of 500 GeV and beyond. One example of this is slepton studies. Sleptons with masses above about 300 GeV will be difficult to find at the LHC, especially if they are not produced copiously in the cascade decays of other strongly-interacting superpartners. Furthermore, if the left- and right-sleptons are close in mass to each other they will be difficult to resolve. The linear collider produces sleptons directly if the CM energy is sufficient. The two species of sleptons are readily distinguished using beam polarization and other observables. Another discovery issue arises in the case of a neutral wino or higgsino LSP, with a nearly degenerate charged $\widetilde{W}^{\pm}$ just above it in mass. The wino case occurs, for example, in anomaly-mediated and in $U(1)$-mediated supersymmetry breaking. In the limit in which all other superpartners are too massive to be produced at the LHC or LC, the linear collider with energy above 500 GeV and 100 fb^{-1} is expected to have a higher mass reach than the LHC for these states. There are other important cases, such as R-parity-violating supersymmetry, in which the linear collider is needed to discover or resolve states of the supersymmetry spectrum.

Supersymmetry and Higgs bosons. The minimal supersymmetric extension of the SM (MSSM) predicts that at least one scalar Higgs boson (h^0) must have mass below about 135 GeV. The mass is controlled at tree-level by the Z-boson mass, and at one loop by the logarithm of superpartner masses. The prediction of a light Higgs boson has two virtues: it is a useful falsifiable test of the MSSM, and fits nicely within the upper bound from the current precision EW data. Over much of the parameter space, the light MSSM Higgs boson behaves very similarly to the SM Higgs boson.

The other physical scalar Higgs states of the MSSM are H^0, A^0, and $H^{\pm}$. Unlike the h^0 state, these Higgs bosons receive tree-level masses directly from supersymmetry breaking parameters. Therefore, it is not possible to rigorously establish upper bounds to their masses. In large parts of parameter space, the masses of these particles are above 300 GeV, and the only important production processes in e^+e^- annihilation are the pair-production reactions $e^+e^- \to H^+H^-, H^0A^0$. Thus, these particles may not appear at the first-stage linear collider.

If the heavy Higgs boson are not seen directly, the effects of the more complicated Higgs sector of the MSSM can be observed by measuring slight deviations in the couplings of h^0 to fermions and gauge bosons from those predicted for a SM Higgs boson. The more massive the heavy Higgs bosons are, the more h^0 behaves like the SM Higgs boson. Nevertheless, inconsistency with the SM can be discerned by precision measurements at the LC over much of the parameter space, even when m_{A^0} is significantly higher than $\sqrt{s}/2$ and out of reach of direct production. This issue is discussed in Chapter 3, Section 8. It demonstrates again the importance of precision Higgs boson measurements to pointing the way to new physics at higher mass scales.

Probing supersymmetry breaking. Finally, precision measurements of supersymmetry masses and mixing angles serve a purpose beyond simply determining what Lagrangian applies to the energy region around the weak interaction scale. Careful measurements can reveal a pattern characteristic of a more fundamental theory. For example, masses measured at the weak scale can be evolved using the renormalization group to a higher scale, where they might be seen to be unified or to fit another simple relation. A pattern that emerged from this study would point to a specific theory of supersymmetry breaking, indicating both the mechanism and scale at which it occurs. This study could also support or refute the hypothesis that our world is derived from a perturbative grand unified theory with an energy desert, a hypothesis that does seem to apply to the precisely known gauge couplings measured at m_Z. The ability of a linear collider to test these tantalizing ideas with precision measurements provides a route by which we can climb from the weak scale to a more profound theory operating at much higher energies.

4 New strong interactions at the TeV scale

A second way to cure the stability crisis of the Higgs field and to explain the origin of electroweak symmetry breaking is to introduce a new set of strong interactions that operate at the TeV scale of energies. In models of this type, symmetry breaking arises in the weak interactions in the same way that it arises in well-studied solid-state physics systems such as superconductors. Just as in those systems, the physics responsible for the symmetry breaking has many other consequences that lead to observable phenomena at the energy scale of the new interactions.

Two quite distinct implementations of this line of thought have been actively pursued. The first follows the possibility that the Higgs doublet (*i.e.*, the four degrees of freedom which after electroweak symmetry breaking become the Higgs boson and the longitudinal components of the $W^\pm$ and Z^0) is a bound state that arises from a short-range strongly coupled force. Theories that have this behavior are generically called 'composite Higgs' models. These models are usually well approximated at low energies by the SM, and therefore are consistent with the electroweak data.

The second implementation follows the possibility that the new strong interactions do not generate a Higgs doublet, even as a bound state. This is possible if the electroweak symmetry is broken by the pair-condensation of some new strongly interacting particles. The prototype of such theories is 'technicolor', an asymptotically-free gauge interaction that becomes strong at the TeV scale. The behavior of technicolor theories below the TeV scale is typically very different from that of the SM. In most cases, there is no Higgs boson with an observable coupling to pairs of Z bosons, and the new symmetry-breaking interactions generate substantial corrections to precision electroweak observables.

The linear collider experiments that directly test these two theoretical pictures are reviewed in detail in Chapter 5, Sections 3 and 4. In this section we briefly discuss the two ideas in general terms and discuss the relevance of the linear collider for uncovering and studying these new interactions.

4.1 Composite Higgs models

Several ways have been suggested in the literature to form a bound-state Higgs boson that mimics the properties of the Higgs particle of the SM. In the top-quark seesaw theory, the Higgs boson arises as a bound state of the left-handed top quark and the right-handed component of a new heavy vector-like quark. Although the composite Higgs boson mass is typically about 500 GeV, there is agreement with the precision electroweak data for a range of parameters in which new contributions from the additional heavy quark compensate the effects of a heavy Higgs boson. Depending on the binding interactions, an extended composite Higgs sector may form. In this case, mixing among the CP-even scalar bound states may bring the SM-like Higgs boson down to a mass below 200 GeV.

Another scenario that may lead to a composite Higgs boson is the SM in extra spatial dimensions, a case that we will discuss in more detail in the next section. Here the short-range strongly-coupled force is given by the Kaluza-Klein excited states of the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge bosons. The Kaluza-Klein states of the top quark become the constituents of the Higgs boson. The Higgs boson in this scenario has a mass of order 200 GeV.

We now list a number of non-standard phenomena that are likely to appear in these theories at relatively low energies. Of course, these theories will ultimately be tested by going to the energy scale of the new interaction and determining its nature as a gauge theory or as a field theory of some other type.

Deviations in Higgs sector. In models in which the Higgs boson appears as a bound state, it is likely that additional composite scalar states will also be present at the TeV scale or below. If these states appear, their masses and couplings will provide important information on the nature of the constituents. Additional states with the quantum numbers of the Higgs boson can be produced at a linear collider in association with a Z^0 or singly in $\gamma\gamma$ collisions. Other states can be studied in pair-production. In both cases, the precise measurement of their masses and branching ratios will provide important information. In addition, it is possible at a linear collider to recognize even very small deviations of the properties of the Higgs boson from the predictions of the SM.

Extra fermions. The top-quark seesaw model implies the existence of an additional fermion whose left- and right-handed components have the same charges as the right-handed top quark, t_R. This quark could have a mass of many TeV with little loss in fine-tuning, making it hard to find directly at any of the next generation colliders, including the LHC. In this circumstance, however, the improved precision electroweak measurements described in Chapter 8 should show a clear deviation from the SM in the direction of positive ρ parameter ($\Delta T > 0$). This would prove that the SM is incomplete and give a clue as to the nature of the new physics.

Heavy vector bosons. Both the top-quark seesaw theory and the extra-dimensional composite Higgs models imply the existence of heavy vector bosons. In the top-condensate scenario, the extra heavy vectors could arise from a topcolor gauge group. In addition, one often requires an additional gauge interaction that couples differently to t_R and b_R to explain why we see top quark but not bottom quark condensation. If a new vector boson couples with some strength to all three generations, it will appear as a resonance at the LHC, and its effects will be seen at the LC as a pattern of deviations in all of the polarized $e^+e^- \to f\overline{f}$ cross sections. In both cases, the experiments are sensitive to masses of 4 TeV and above. This mass reach overlaps well with the expectation that the new physics should occur at a mass scale of several TeV. The observation and characterization of new Z bosons are described in Chapter 5, Section 5.

4.2 Technicolor theories

Technicolor theories provide an alternative type of model with new strong interactions. These theories do not require a composite Higgs boson. Instead, they involve new chiral fermions and a confining gauge interaction that becomes strongly-coupled at an energy scale of order 1 TeV. The most robust prediction of these theories is that there is a vector resonance with mass below about 2 TeV that couples with full strength to the $J = 1$ W^+W^- scattering amplitude.

The general idea of technicolor is severely constrained by the precision electroweak measurements, which favor models with a light Higgs boson over models where this state is replaced by heavy resonances. In order to be viable, a technicolor model must provide some new contributions to the precision electroweak observables that compensate for the absence of the Higgs boson. This leads us away from models in which the new strong interactions mimic the behavior of QCD and toward models with a significantly different behavior. For such models, it is difficult to compute quantitatively and so we must look for qualitative predictions that can be tested at high-energy colliders. In this situation, the ability of the linear collider to discover new particles essentially independently of their decay schemes would play an important role.

We summarize some of the measurements that the linear collider can perform that are relevant to strongly-coupled theories of this type. Our approach is to identify qualitative features that are likely to result from technicolor dynamics. Because of the uncertainties in calculating the properties of such strongly-interacting theories, it is not possible to map out for what parameters a given model can be confirmed or ruled out. Nevertheless, the linear collider has the opportunity to identify key components of technicolor models.

Strong WW scattering. As we have noted, the most robust qualitative prediction of technicolor theories is the presence of a resonance in WW scattering in the vector ($J = 1$) channel. This particle is the analogue of the ρ meson of QCD. For masses up to 2 TeV, the 'techni-ρ' should be seen as a mass peak in the W^+W^- invariant mass distribution observed at the LHC. In addition, the techni-ρ will appear as a resonance in $e^+e^- \to W^+W^-$ for longitudinal W polarizations, for the same reason that in QCD the ρ meson appears as a dramatic resonance in $e^+e^- \to \pi^+\pi^-$. The resonant effect is a very large enhancement of a well-understood SM process, so the effect should be unmistakable at the linear collider, even at $\sqrt{s} = 500$ GeV, well below the resonance. As with the case of a Z', the two different observations at the linear collider and the LHC can be put together to obtain a clear phenomenological picture of this new state. These issues are discussed further in Chapter 5, Section 3.

Anomalous gauge couplings. If there is no Higgs boson resonance below about 800 GeV, the unitarization of the $WW \to WW$ scattering cross-section by new strong interactions will lead to a large set of new effective interactions that alter the couplings of W and Z. Some of these terms lead to anomalous contributions to the $WW\gamma$ and WWZ vertices. Through the precision study of $e^+e^- \to W^+W^-$ and related reactions, the 500 GeV linear collider with 500 fb^{-1} of integrated luminosity will detect these anomalous contributions or improve the limits by a factor of ten over those that will be set at the LHC. In the case that there are new strong interactions, the accuracy of the linear collider measurement is such as to make it possible to measure the coefficients of the effective Lagrangian that results from the new strong interactions. These measurements are discussed further in Chapter 5, Section 2. In addition, many technicolor models predict large anomalous contributions to the gauge interactions of the top quark particularly to the $t\bar{t}Z$ vertex function. The linear collider may provide the only way to measure this vertex precisely. The measurement is discussed in Chapter 6, Section 3.

Extra scalars. Just as, in QCD, where the strongly coupled quarks lead to octets of relatively light mesons, technicolor theories often imply the existence of a multiplet of pseudoscalar bosons that are relatively light compared to the TeV scale. These bosons are composites of the underlying strongly coupled fermions. Since these particles have non-zero electroweak quantum numbers, they are pair-produced in e^+e^- annihilation. The number of such bosons and their quantum numbers depend on the precise technicolor theory. Experimentally, these particles look like the particles of an extended Higgs sector, and their detection and study follow the methods discussed for that case in Chapter 2, Section 6. Particular models may include additional new particles. For example, in 'topcolor-assisted technicolor', there is a second doublet of Higgs bosons, with masses of 200-300 GeV, associated with top-quark mass generation.

5 Extra spatial dimensions

It is 'apparent' that the space we live in is three-dimensional, and in fact precise measurements are consistent with this even down to the small distances probed by LEP2 and the Tevatron. But one should not hastily conclude that the universe has no more than three dimensions, because two important loopholes remain. First, there could be extra spatial dimensions that are not accessible to SM particles such as the photon and the gluon. Second, there could be extra spatial dimensions that are compact, with a size smaller than 10^{-17} cm. In both cases, it is possible to build models that are in agreement with all current data.

Besides being a logical possibility, the existence of extra spatial dimensions may explain key features of observed phenomena, ranging from the weakness of the gravitational interactions to the existence of three generations of quarks and leptons. Most importantly from the viewpoint of the stability problem of the Higgs field, the assumption that the universe contains more that three dimensions opens a number of new possibilities for models of electroweak symmetry breaking. In such models, the value of the weak-interaction scale results from the fact that some natural mass scale of gravity in higher dimensions, either the size of the new dimensions or the intrinsic mass scale of gravity, is of order 1 TeV. This, in turn, leads to new observable phenomena in high energy physics at energies near 1 TeV. These phenomena, and the possibility of their observation at a linear collider, are discussed in Chapter 5, Section 6.

Once we have opened the possibility of new spatial dimensions, there are many ways to construct models. Most of the options can be classified by two criteria. First, we must specify which particles are allowed to propagate in the full space and which are restricted by some mechanism to live in a three-dimensional subspace. Second, we must specify whether the extra dimensions are flat, like the three dimensions we see, or highly curved. The latter case is referred to in the literature as a 'warped' geometry. Some ideas may require additional fields, beyond the SM fields, to solve certain problems (such as flavor violation or anomaly cancelation) that can arise from the hypothesized configuration of particles in the extra-dimensional space. We now give a brief overview of these possibilities and the role of the linear collider in each scenario.

5.1 Flat extra dimensions, containing only gravity

The first possibility is that all of the particles of the SM—quarks, leptons, and Higgs and gauge bosons—are localized on three-dimensional walls ('3-branes') in a higher-dimensional space. Gravity, however, necessarily propagates through all of space. Higher-dimensional gravity can be described in four-dimensional terms by using a momentum representation in the extra dimensions. If these extra dimensions are compact, the corresponding momenta are quantized. Each possible value of the extra-dimensional momentum gives a distinct particle in four dimensions. This particle has mass $m_i^2 = (\vec{p}_i)^2$, where $\vec{p}_i$ is the quantized value of the extra-dimensional momenta. These four-dimensional particles arising from a higher-dimensional field are called Kaluza-Klein (KK) excitations. In the later examples, where we put SM fields also into the higher dimensions, these field will also acquire a KK spectrum.

If gravity propagates in the extra dimensions, the exchange of its KK excitations will increase the strength of the gravitational force at distances smaller than the size of the new dimensions. Then the fundamental mass scale M_* at which gravity becomes a strong interaction is lower than the apparent Planck scale of 10^{19} GeV. It is possible that M_* is as low as 1 TeV if the volume of the extra dimensions is sufficiently large. In that case, there is no stability problem for the Higgs field. The Higgs expectation value is naturally of the order of M_*.

The KK gravitons can be produced in collider experiments. In e^+e^- collisions, one would look for e^+e^- annihilation into a photon plus missing energy. The cross section for this process has typical electroweak size as the CM energy approaches M_* and the phase space for producing the KK gravitons opens up. The expected signals of extra dimensions are highly sensitive to the number of extra dimensions. Nevertheless, if the number of extra dimensions is less than or equal to six, the signal can be studied at a linear collider at CM energies that are a factor of 3–10 below M_*. The LHC can also study KK graviton production through processes such as $q\overline{q}$ annihilation to a jet plus missing energy. The sensitivity to M_* is somewhat greater than that of a 1 TeV linear collider, but it is not possible to measure the missing mass of the unobserved graviton.

The KK gravitons can also appear through their virtual exchange in processes such as $e^+e^- \to f\overline{f}$, $e^+e^- \to \gamma\gamma$, and $e^+e^- \to gg$. The graviton exchange leads to a spin-2 component that is distinct from the SM expectation. Although this indirect signal of KK gravitons is more model-dependent, it is expected that it can be seen even at 500 GeV if M_* is less than a few TeV.

5.2 Warped extra dimensions, containing only gravity

If the extra dimensions are warped, the KK spectrum of gravitons has somewhat different properties. In the case of flat extra dimension, the KK particles are closely spaced in mass, but in the case of warped dimensions, the spacing is of order 1 TeV. In the simplest model, the KK gravitons have masses in a characteristic pattern given by the zeros of a Bessel function. The individual states appear as spin-2 resonances coupling with electroweak strength to e^+e^- and $q\overline{q}$. These resonances might be seen directly at the LHC or at a linear collider. If the resonances are very heavy, their effects can be seen from additional spin-2 contact contributions to $e^+e^- \to f\overline{f}$, even for masses more than an order of magnitude above the collider CM energy.

5.3 Flat extra dimensions, containing SM gauge fields

It is often assumed that the quarks and leptons are localized on three-dimensional walls (3-branes) and therefore do not have KK modes, whereas the gauge bosons propagate in the extra-dimensional space. In this case, the KK modes of the electroweak gauge bosons contribute at tree level to the electroweak observables, so that a rather tight lower bound of about 4 TeV can be imposed on the inverse size of the extra dimensions. The LHC should be able to see the first gauge boson KK resonance up to about 5 TeV, leaving a small window of available phase space for direct production of these states. On the other hand, precision measurements at a high-energy e^+e^- linear collider can establish a pattern of deviations from the SM predictions for the reactions $e^+e^- \to f\overline{f}$ from KK resonances well beyond direct production sensitivities. The capability of an e^+e^- linear collider in identifying the rise in cross sections due to KK resonances improves when the center-of-mass energy is increased. High luminosity is also important. For example, with more than 100 fb^{-1} of integrated luminosity at a 500 GeV, one could see the effects of resonance tails for KK masses above 10 TeV in models with one extra dimension.

5.4 Flat extra dimensions, containing all SM particles

Finally, we consider the case of 'universal' extra dimensions, in which all SM particles are permitted to propagate. A distinctive feature of universal extra dimensions is that the quantized KK momentum is conserved at each vertex. Thus, the KK modes of electroweak gauge bosons do not contribute to the precision electroweak observables at the tree level. As a result, the current mass bound on the first KK states is as low as 300 GeV for one universal extra dimension. If the KK states do indeed have a mass in the range 300-400 GeV, we would expect to observe the states at the Tevatron and the LHC. The linear collider, at a CM energy of 800 GeV, would become a KK factory that produces excited states of quarks, leptons, and gauge bosons.

6 Surprises

Our brief discussion of pathways beyond the SM concentrated on three very different approaches that have been proposed to solve the conundrums of the SM. Although some of these ideas are more easily tested than others at the next-generation colliders, it is important to note that all three approaches have many new observable consequences. In all cases, we expect to see an explosion of new phenomena as we head to higher energies.

Though these three approaches are very different, we should not delude ourselves into thinking that they cover the full range of possibilities. Letting our imaginations run free, we could envision models in which quantum field theory itself breaks down at the weak interaction scale and an even more fundamental description takes over. Such a possibility would be viable only if it satisfies the constraint of giving back the predictions of the SM at energies below 100 GeV. String theory is an example of a framework that resembles the SM at low energies but, at the energies of the string scale, is dramatically different from a simple quantum field theory. Perhaps there are other alternatives to be found.

Exploring physics at shorter distances and with higher precision is an endeavor that implies the possibility of great surprises. Experiments at a linear collider will be a necessary and rewarding part of this program, and will constitute a major step in our quest to understand how Nature works.