Precise predictions for the neutral Higgs boson masses in the MSSM

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Sitges, 04/99

based on collaboration with
S. Heinemeyer and *W. Hollik*

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1. Introduction

Stringent direct test of SUSY:

Light Higgs boson h required

m_h calculable from MSSM parameters:

Tree level: $m_h < M_Z$

Large radiative corrections: $\Rightarrow$ $m_h \lesssim 135$ GeV

$\Rightarrow$ Precise prediction needed for Higgs search: Discovery/exclusion potential of LEP2 and upgraded Tevatron

High-precision measurement of m_h at LHC/LC: Sensitive test of MSSM

Precise prediction for m_H:
$\Rightarrow$ Resolution of splitting m_H, M_A

Higgs sector of the MSSM:

MSSM: Enlarged Higgs sector:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

Physical states: $h^0, H^0, A^0, H^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

In SUSY: m_h is not a free parameter!

Tree-level mass matrix (in $\phi_1 - \phi_2$ basis):

$$M^{2,\mathrm{tree}}_\mathrm{Higgs} = \begin{pmatrix} m^2_{\phi_1} & m^2_{\phi_1\phi_2} \\ m^2_{\phi_1\phi_2} & m^2_{\phi_2} \end{pmatrix} =$$

$$\begin{pmatrix} M_\mathrm{A}^2 \sin^2\beta + M_\mathrm{Z}^2 \cos^2\beta & -(M_\mathrm{A}^2 + M_\mathrm{Z}^2)\sin\beta\cos\beta \\ -(M_\mathrm{A}^2 + M_\mathrm{Z}^2)\sin\beta\cos\beta & M_\mathrm{A}^2 \cos^2\beta + M_\mathrm{Z}^2 \sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \quad \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m^{2,\mathrm{tree}}_\mathrm{H} & 0 \\ 0 & m^{2,\mathrm{tree}}_\mathrm{h} \end{pmatrix}$$

$\Rightarrow$ m_h, m_H, mixing angle α at tree level

Tree-level result for m_h:

$$m_\mathrm{h}^2 = \frac{1}{2}\left[M_\mathrm{A}^2 + M_\mathrm{Z}^2 - \sqrt{(M_\mathrm{A}^2 + M_\mathrm{Z}^2)^2 - 4M_\mathrm{Z}^2 M_\mathrm{A}^2 \cos^2 2\beta}\right]$$

$\Rightarrow$ $m_\mathrm{h} \leq M_\mathrm{Z}$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Radiative corrections to the mass of the lightest Higgs boson:

Mainly used up to now for phenomenological applications:

- RG results
  Logarithmic corrections to Higgs potential, resummation of leading logs
  [*J. Casas, J. Espinosa, M. Quirós, A. Riotto '95*]
  [*M. Carena, M. Quiros, C. Wagner '95*]
  [*H. Haber, R. Hempfling, A. Hoang '97*]

- Complete diagrammatic one-loop results in on-shell scheme
  Explicit Feynman-diagrammatic calculation, Higgs mass: pole of propagator
  [*P. Chankowski, S. Pokorski, J. Rosiek '94*]
  [*A. Dabelstein '95*]
  [*D. Pierce, J. Bagger, K. Matchev, R. Zhang '97*]

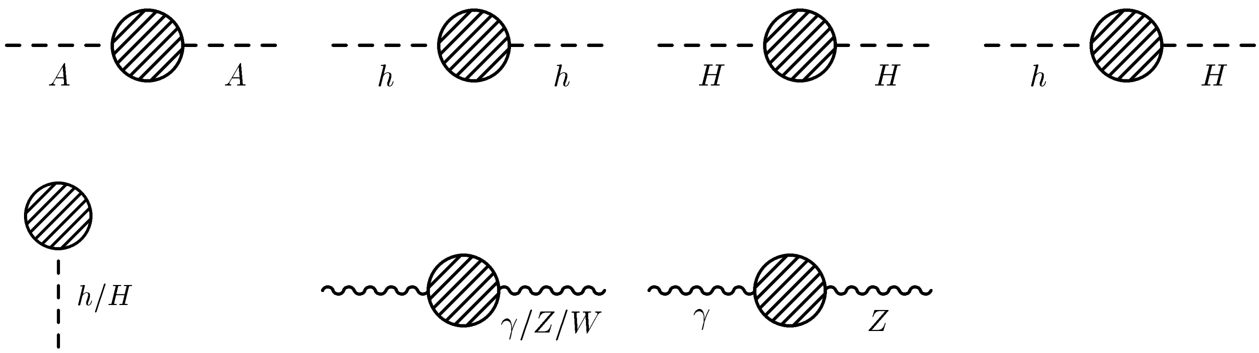
Differ by $\mathcal{O}(10\,\text{GeV})$
Not easy to compare: difference in non-leading one-loop contributions and higher-order logs

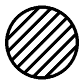
$\Rightarrow$ Diagrammatic two-loop results desirable for precise prediction of Higgs-boson masses

2. Diagrammatic two-loop calculation of the masses of the neutral $\mathcal{CP}$-even Higgs bosons

Corrections to h/H propagators $\Rightarrow m_{\mathrm{h}}, m_{\mathrm{H}}$

On-shell renormalization: M_i: physical masses





: Complete MSSM in 1-loop order

[A. Dabelstein '95]

$+\ \mathcal{O}(\alpha\alpha_s)$ contributions from $t-\tilde{t}$-sector (Yukawa contribution, $p^2 = 0$)

$+\ G_\mu^2 m_{\mathrm{t}}^6$ improvement terms

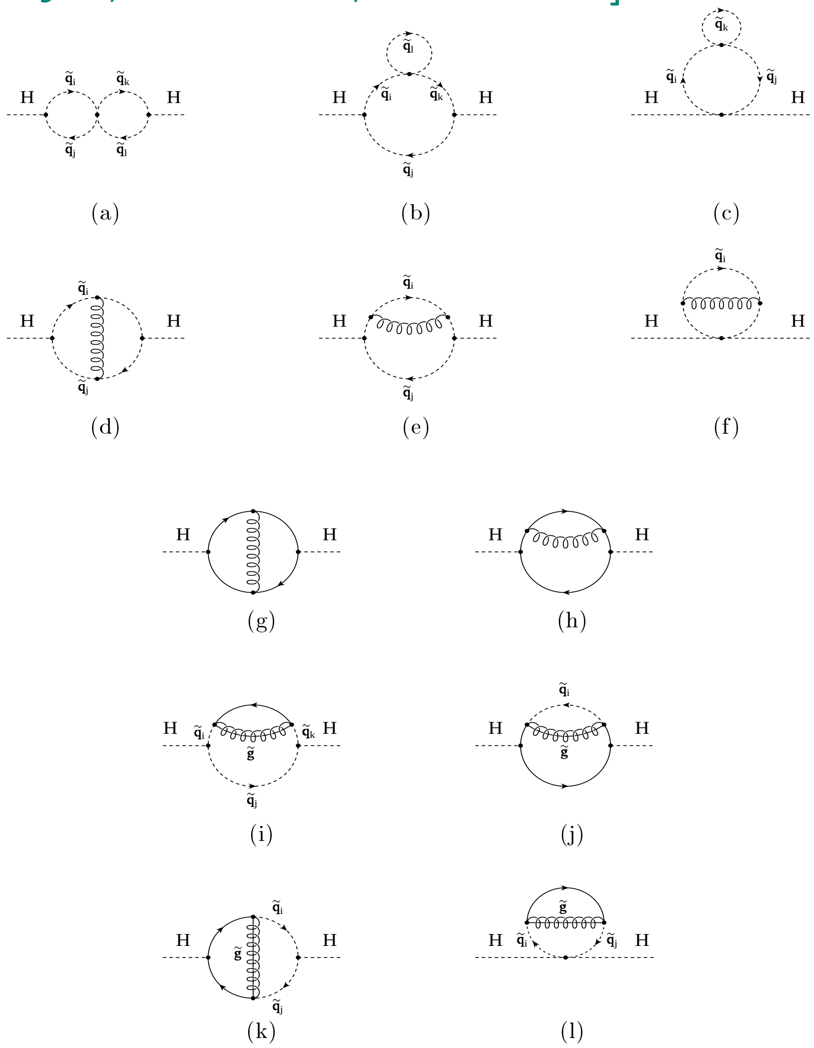
[M. Carena, J. Espinosa, M. Quirós, C. Wagner '95]

Physical masses m_{h}^2, m_{H}^2:

poles of h/H propagators $\Rightarrow$ solutions of

$$\left[q^2 - m_{\mathrm{h}}^2 + \hat{\Sigma}_{\mathrm{hh}}(q^2)\right]\left[q^2 - m_{\mathrm{H}}^2 + \hat{\Sigma}_{\mathrm{HH}}(q^2)\right] - \left(\hat{\Sigma}_{\mathrm{hH}}(q^2)\right)^2 = 0.$$

Leading two-loop corrections:
$\mathcal{O}(\alpha\alpha_s)$ contribution from $t - \tilde{t}$-sector
(Yukawa contribution, $p^2 = 0$)
[*S. Heinemeyer, W. Hollik, G. W. '98*]



$\tilde{t}$ sector:

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_1 & m_t M_t^{LR} \\ m_t M_t^{LR} & M^2_{\tilde{t}_R} + m_t^2 + DT_2 \end{pmatrix}$$

$M_t^{LR} = A_t - \mu \cot\beta$; large mixing possible

$\Rightarrow$ Physical parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

Large Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}$, $\frac{e\, m_t^2}{M_W s_W}$, ...

3. Compact formula for leading contributions

Goal: Extract dominant contributions from full result $\Rightarrow$ Simple analytical expression

- Trace source of largest corrections
- Handy approximation formula with sufficient accuracy: easy numerical implementation, very high speed

Contribution from t–$\tilde{t}$ sector up to $\mathcal{O}(\alpha\alpha_s)$:

Expansion in

$$\Delta_{\tilde{t}} = \frac{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2} + m^2_{\tilde{t}_1}} = \frac{m_t |M_t^{LR}|}{M_S^2}$$

$$M_S = \sqrt{m^2_{\tilde{q}} + m^2_t} \qquad \text{for } M_{\tilde{t}_L} = M_{\tilde{t}_R} = m_{\tilde{q}}$$

$$M_S = \left(M^2_{\tilde{t}_L} M^2_{\tilde{t}_R} + m^2_t (M^2_{\tilde{t}_L} + M^2_{\tilde{t}_R}) + m^4_t\right)^{\frac{1}{4}}$$

$$\text{for } M_{\tilde{t}_L} \neq M_{\tilde{t}_R}$$

Remaining 1-loop contribution: $\Delta m_h^{2,(1),\mathrm{LLE}}$
leading log result [*H. Haber, R. Hempfling '93*]

assumed: $m_{\tilde{g}} = M = M_{\mathrm{SUSY}} = \sqrt{M_S^2 - m_t^2}, \; \ldots$

Approximation formula for m_{h}:

$$
\begin{aligned}
m_{\mathrm{h}}^2 = {} & \Delta m_{\mathrm{h}}^{2,\mathrm{tree}} \\
& + \Delta m_{\mathrm{h}}^{2,(1),\tilde{t}} + \Delta m_{\mathrm{h}}^{2,(1),\mathrm{LLE}} + \Delta m_{\mathrm{h}}^{2,(2),G_\mu^2} \\
& - \frac{G_\mu \sqrt{2}}{\pi^2} \frac{\alpha_s}{\pi} \overline{m}_{\mathrm{t}}^4 \left[3 \log^2 \left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2} \right) + 2 \log \left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2} \right) + 4 \right. \\
& - 6 \frac{M_t^{LR}}{M_{\mathrm{S}}} - \frac{\left(M_t^{LR}\right)^2}{M_{\mathrm{S}}^2} \left\{ 3 \log \left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2} \right) + 8 \right\} \\
& \left. + \frac{17}{12} \frac{\left(M_t^{LR}\right)^4}{M_{\mathrm{S}}^4} \right] \left(1 + 4 \frac{M_{\mathrm{Z}}^2}{M_{\mathrm{A}}^2} \cos^2\beta \cos 2\beta \right).
\end{aligned}
$$

$$
\overline{m}_{\mathrm{t}}(m_{\mathrm{t}}) = m_{\mathrm{t}} / (1 + \tfrac{4}{3\pi} \alpha_{\mathrm{s}}(m_{\mathrm{t}}))
$$

Maxima/minimum at 1-loop/2-loop :

$$
\frac{M_t^{LR}}{m_{\tilde{q}}} = \begin{cases}
\sqrt{6} - \frac{\alpha_s}{\pi} \left[-1 + 3\sqrt{6} - \sqrt{6} \log \left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2} \right) \right] & \text{(Max)} \\
0 - 2 \frac{\alpha_s}{\pi} & \text{(Min)} \\
-\sqrt{6} + \frac{\alpha_s}{\pi} \left[1 + 3\sqrt{6} - \sqrt{6} \log \left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2} \right) \right] & \text{(Max)}
\end{cases}
$$

$\Rightarrow$ Maxima shifted from $\approx \pm 2.4$ to $\approx \pm 2$

4. The programs FeynHiggs and FeynHiggsFast

[*S. Heinemeyer, W. Hollik, G. W. '98, '99*]

FeynHiggs:

- Implementation of diagrammatic two-loop result into Fortran code:

  Complete $\mathcal{O}(\alpha)+\mathcal{O}(\alpha\alpha_s)+$ lead. high. order

- $\approx$ 60.000 lines Fortran code (mostly $\hat{\Sigma}^{(2)}_{\phi_2}(0)$)
  3.8 MB executable file

- Runtime: ca. 0.5 seconds

- Input:
  $m_{\mathrm{t}}, \ldots,\ \tan\beta,\ M_{\mathrm{A}},\ \mu$
  $m_{\tilde{\mathrm{t}}_1},\ m_{\tilde{\mathrm{t}}_2},\ \theta_{\tilde{\mathrm{t}}} \longleftrightarrow M_{\tilde{t}_L},\ M_{\tilde{t}_R},\ M_t^{LR}$
  $m_{\tilde{\mathrm{g}}},\ M\ \ (M \equiv M_2,\ M_1 = \frac{5}{3}\frac{s_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}M)$

FeynHiggsFast:

- Compact approximation formula

- $\approx$ 1400 lines Fortran code
  65 KB executable file

- Runtime: ca. 2×10^{-5} seconds

- Accuracy: better than 2 GeV for most parts of MSSM parameter space

Contained in both programs:

$\mathcal{O}(\alpha\alpha_s)$ result for $\Delta\rho$ in the MSSM:

[*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97, '98*]

Constraint from $\Delta\rho$: $\Delta\rho^{\text{SUSY}} \lesssim 1 \cdot 10^{-3}$

$\Rightarrow$ Constraints on $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$

Homepage for both programs:

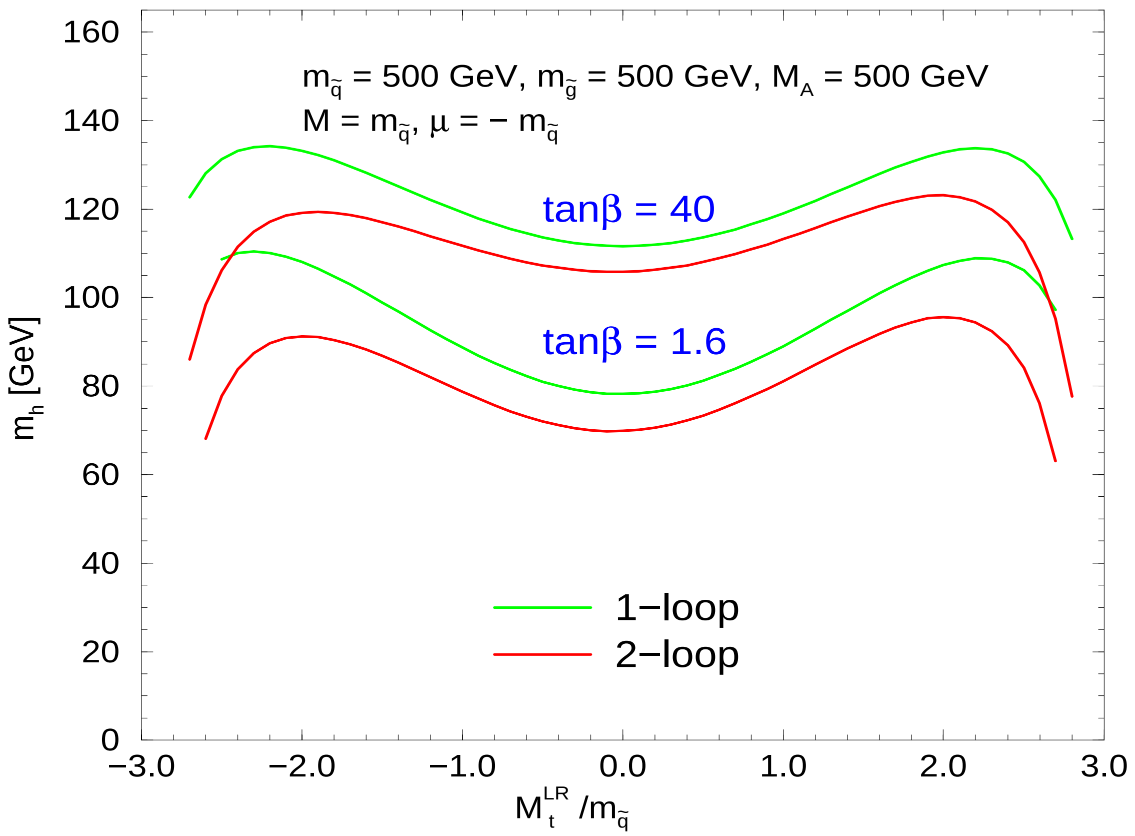
`www-itp.physik.uni-karlsruhe.de/feynhiggs`

5. Numerical Results for $m_{\rm h}$

In following plots: $m_{\tilde{q}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$
$m_{\rm t} = 175$ GeV, $\alpha_{\rm s}(m_{\rm t}) = 0.1095$

Of special interest for testing GUT scenarios:
$\tan\beta \sim 1.6$, $\tan\beta \sim 40$

Dependence on mixing in $\tilde{t}$ sector:



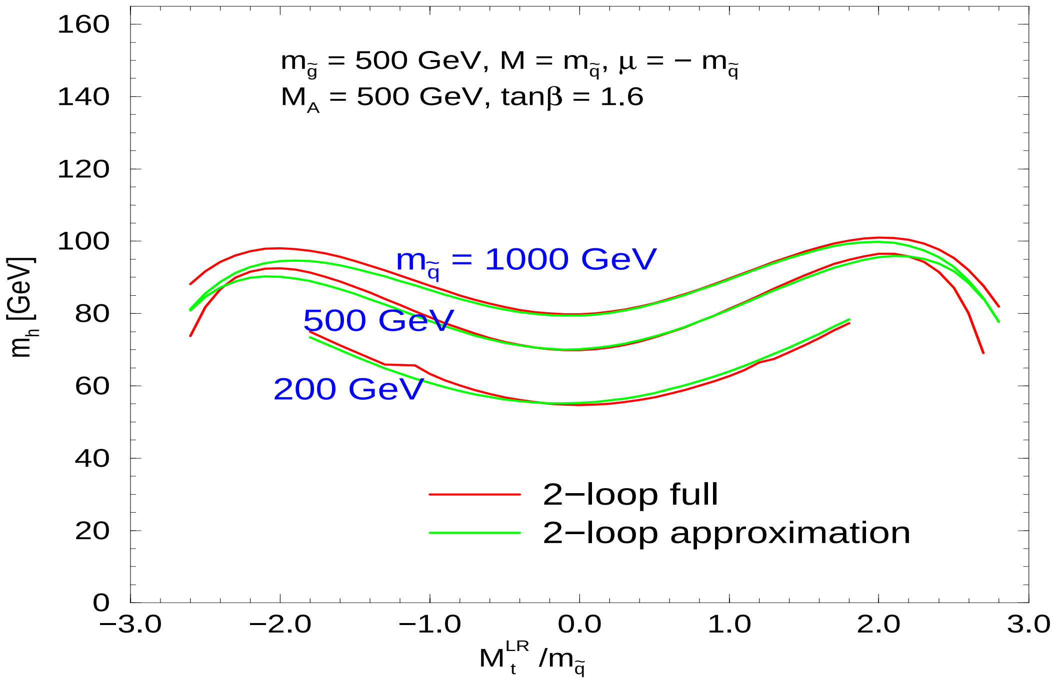
$\Rightarrow$ $M_t^{LR} \approx 0$: "no mixing"

$M_t^{LR} \approx \pm 2\, m_{\tilde{q}}$: "maximal mixing"

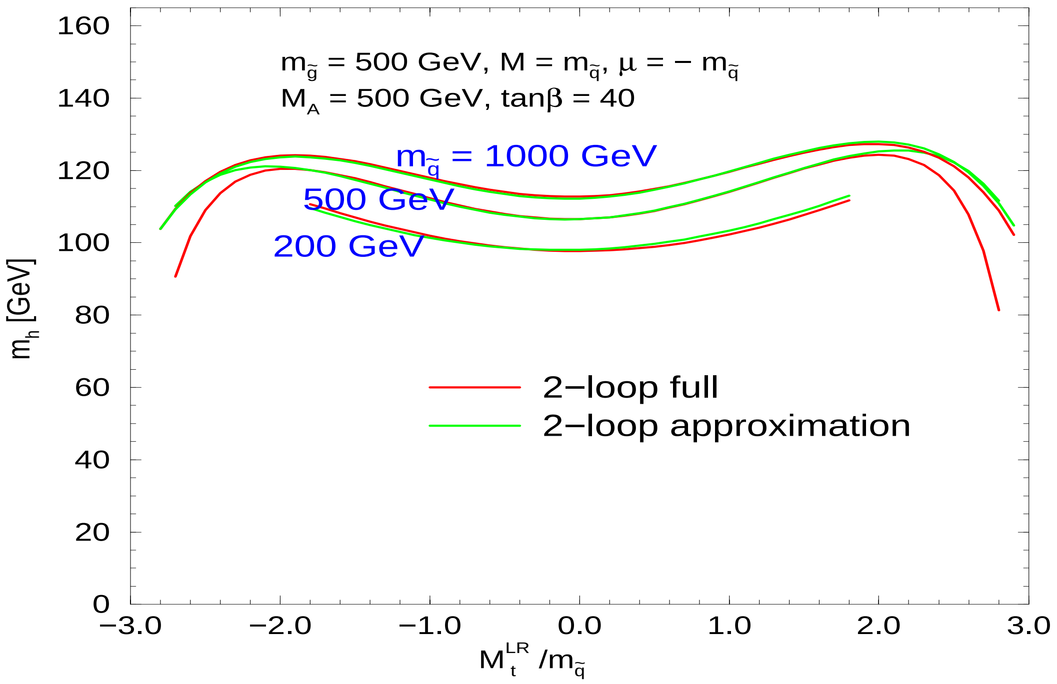
Maximum shifted compared to one-loop result,
one-loop: $M_t^{LR} \approx \pm 2.4\, m_{\tilde{q}}$

Comparison of approximation formula with full result:

Dependence on mixing in $\tilde{t}$ sector, $\tan\beta = 1.6$:



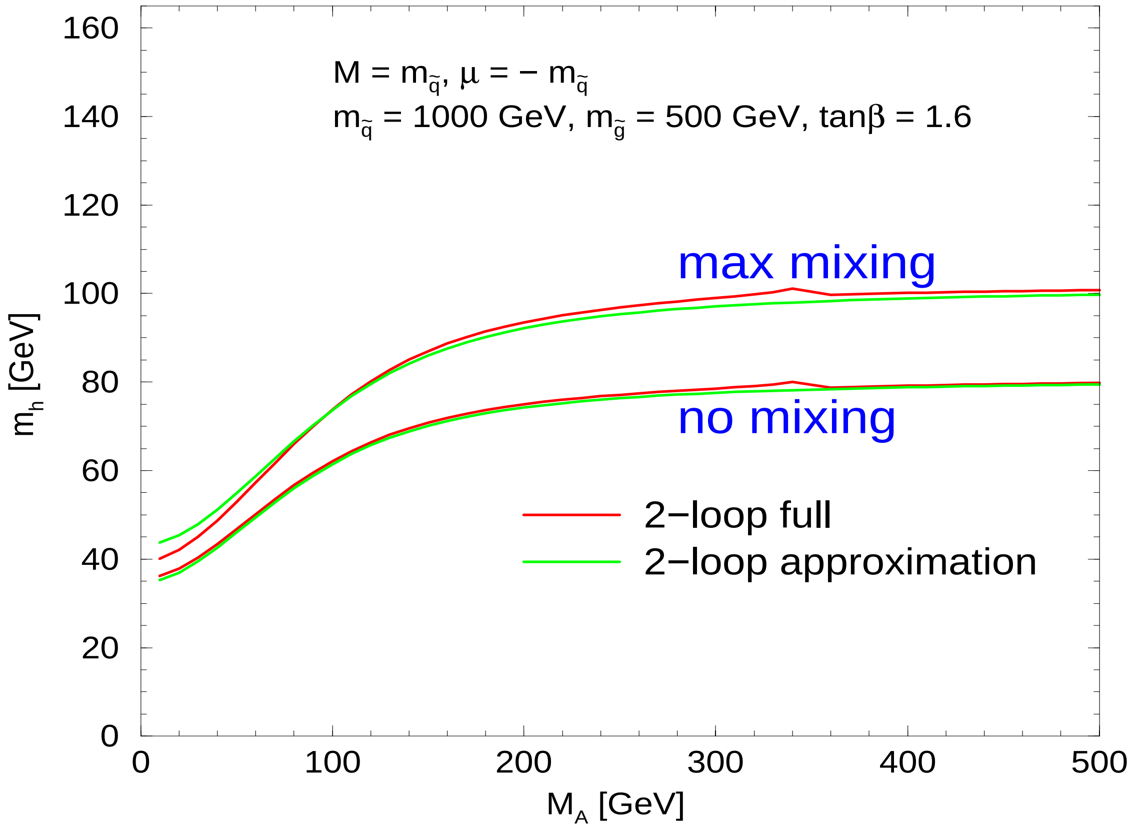
$\tan\beta = 40$:



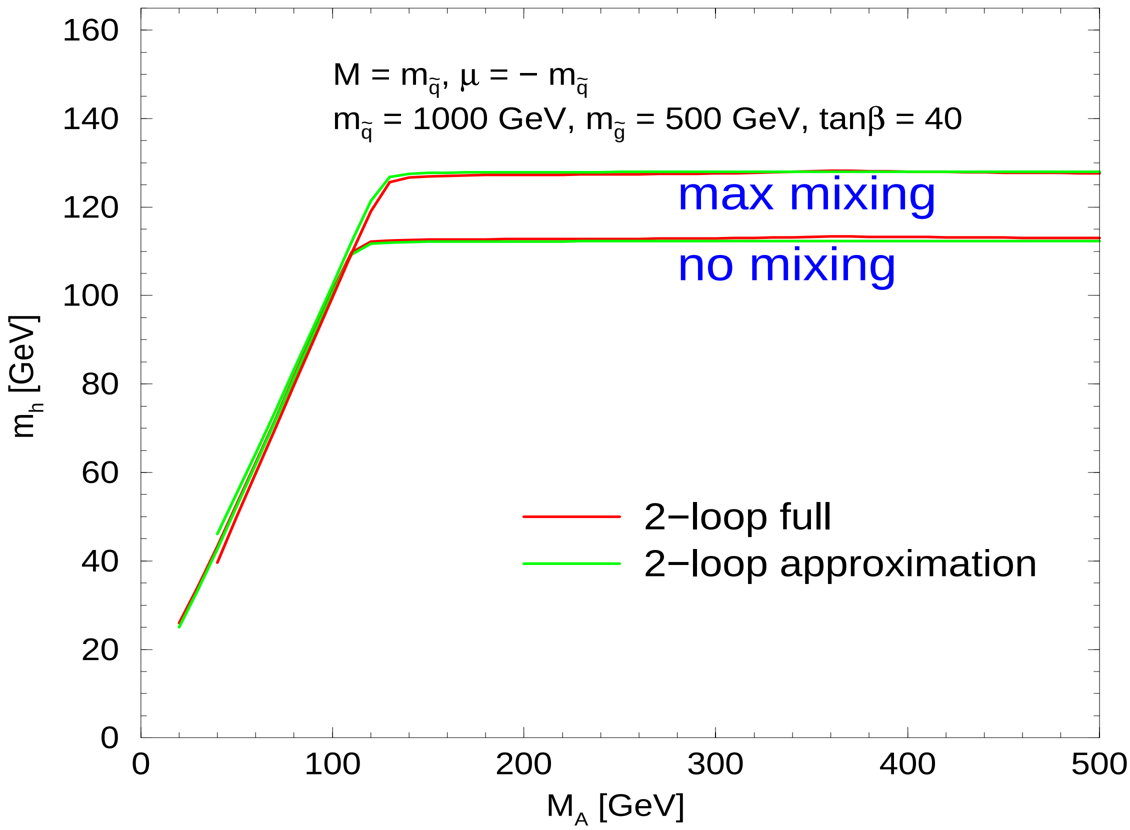
$\Rightarrow$ Very good agreement within ≈ 2 GeV for most of the parameter space

Dependence on M_A :

For $\tan\beta = 1.6$:

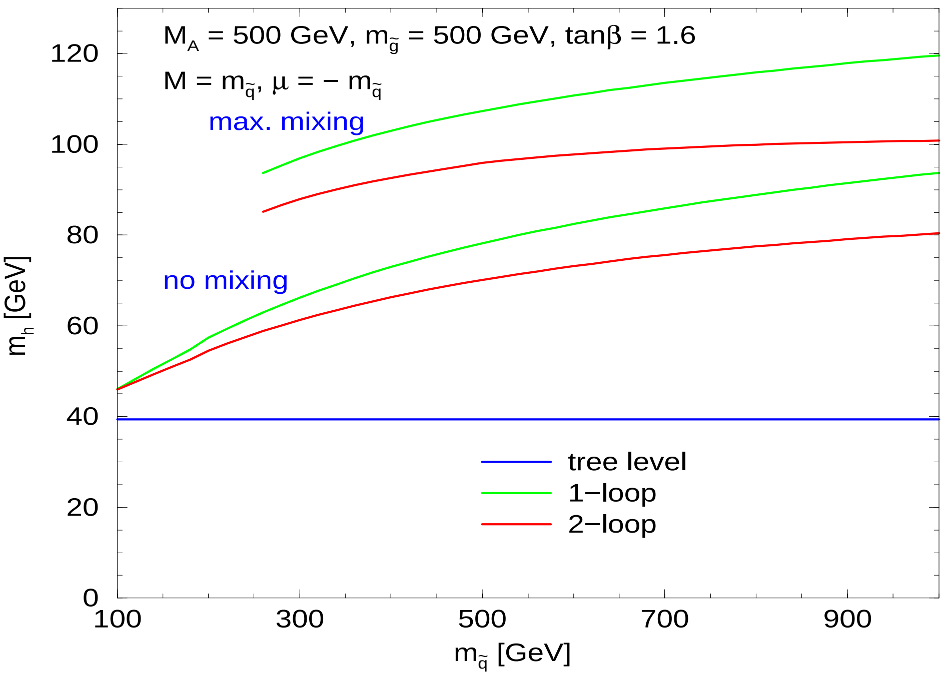


For $\tan\beta = 40$:

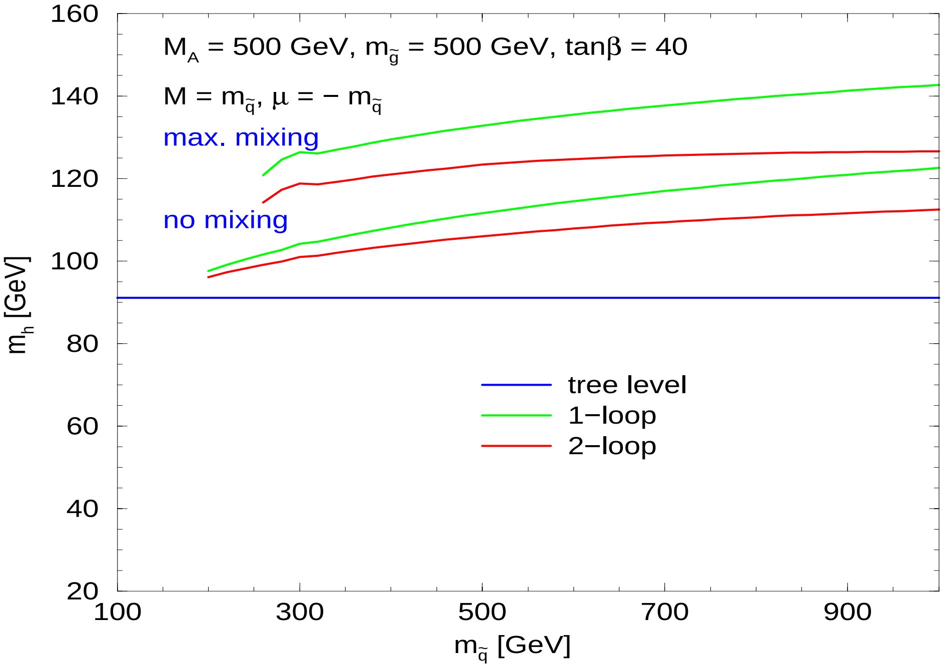


Dependence on $m_{\tilde{q}}$:

For $\tan\beta = 1.6$:



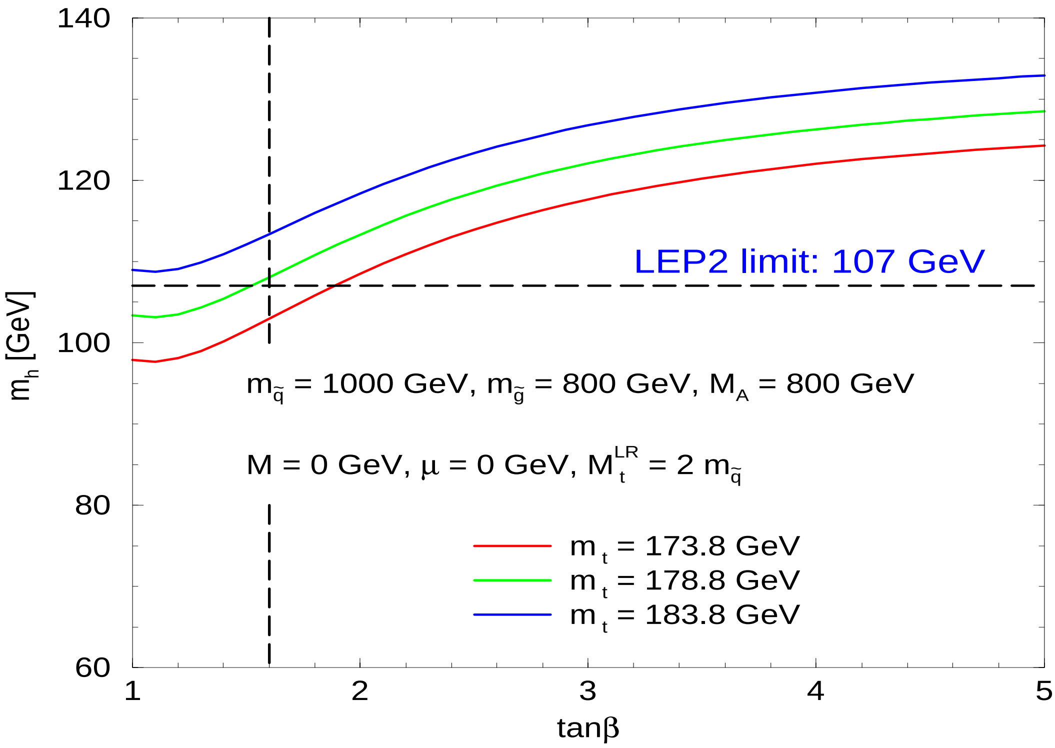
For $\tan\beta = 40$:



$\Rightarrow$ Large correction to one-loop contribution
Considerable reduction of m_h value

Maximal value of m_h in the MSSM?

Maximal m_h-value in scenario with small $\tan\beta$, $m_{\tilde{q}} = 1000$ GeV for different values of m_t:
$m_t = m_t^{exp}$, $m_t^{exp} + 1\sigma$, $m_t^{exp} + 2\sigma$



Values increase by 3–4 GeV for $m_{\tilde{q}} = 2000$ GeV

LEP2 exclusion limit: ≈ 107 GeV

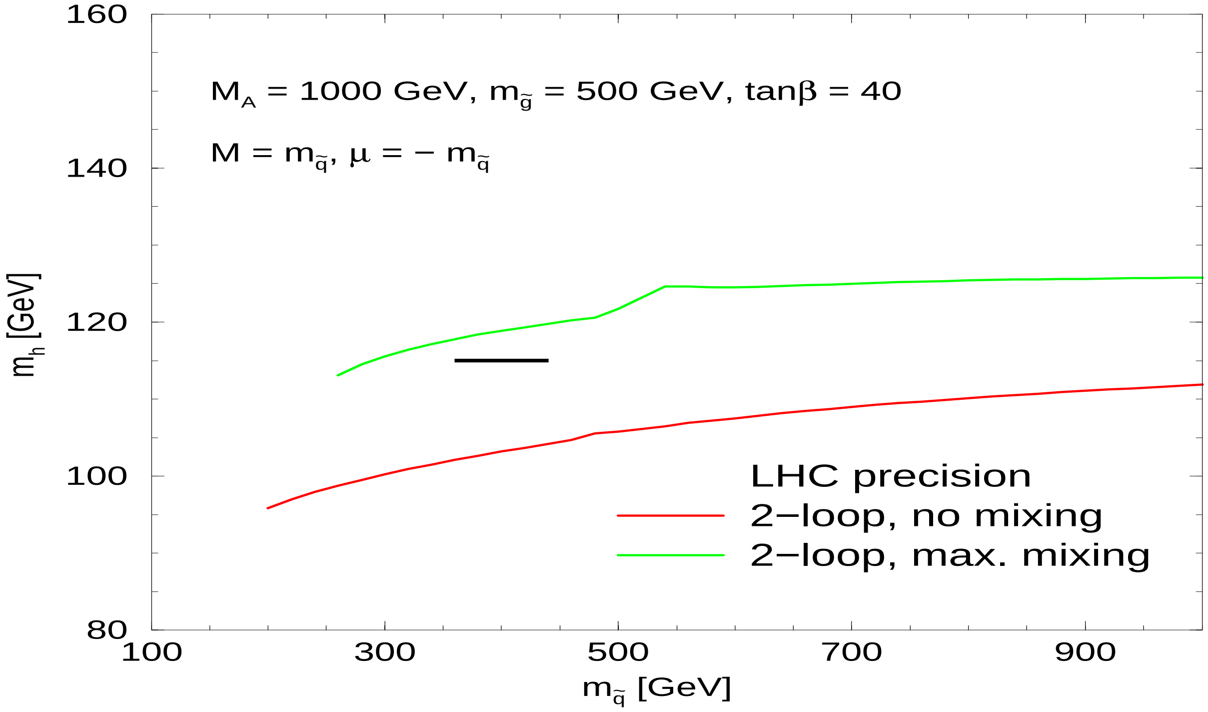
$\Rightarrow$ m_h^{max} for $\tan\beta = 1.6$ is at the edge of LEP2 reach

$\tan\beta$ reach: $\tan\beta = 1.9$ for $m_t = 173.8$ GeV
$\tan\beta = 1.5$ for $m_t = 178.8$ GeV
for $m_{\tilde{q}} = 1000$ GeV

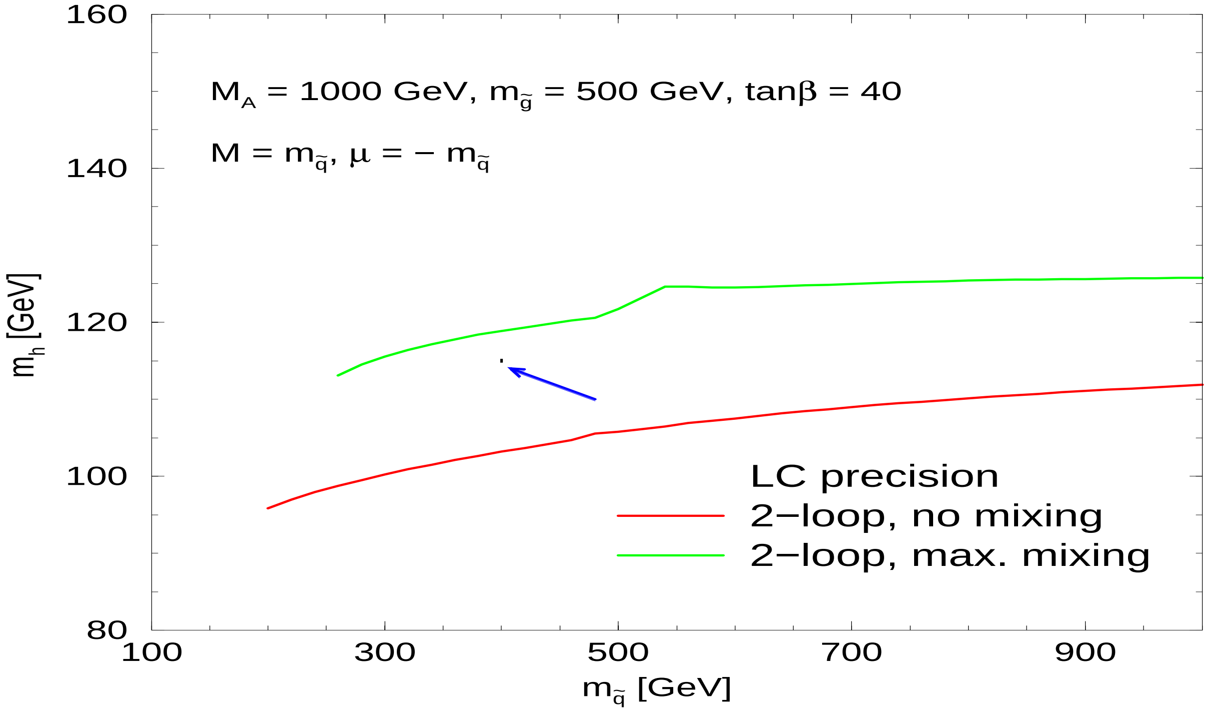
LHC/LC: Experimental information on m_h, M_{SUSY}

Assume: $m_h^{exp} = 115$ GeV, $M_{SUSY}^{exp} = 400$ GeV

LHC results vs. MSSM prediction for m_h:



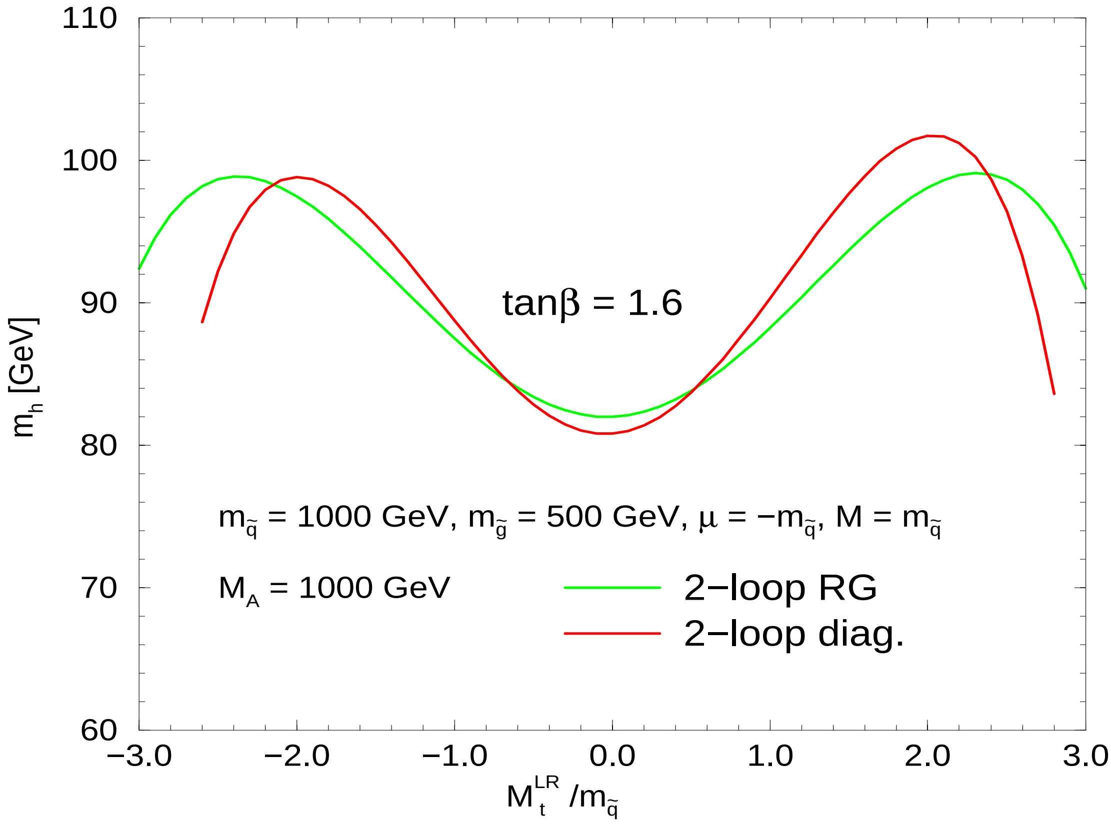
LC results vs. MSSM prediction for m_h:



⇒ Stringent test of MSSM

6. Comparison with RG approach

Dependence on mixing in $\tilde{t}$ sector:



Different shape:

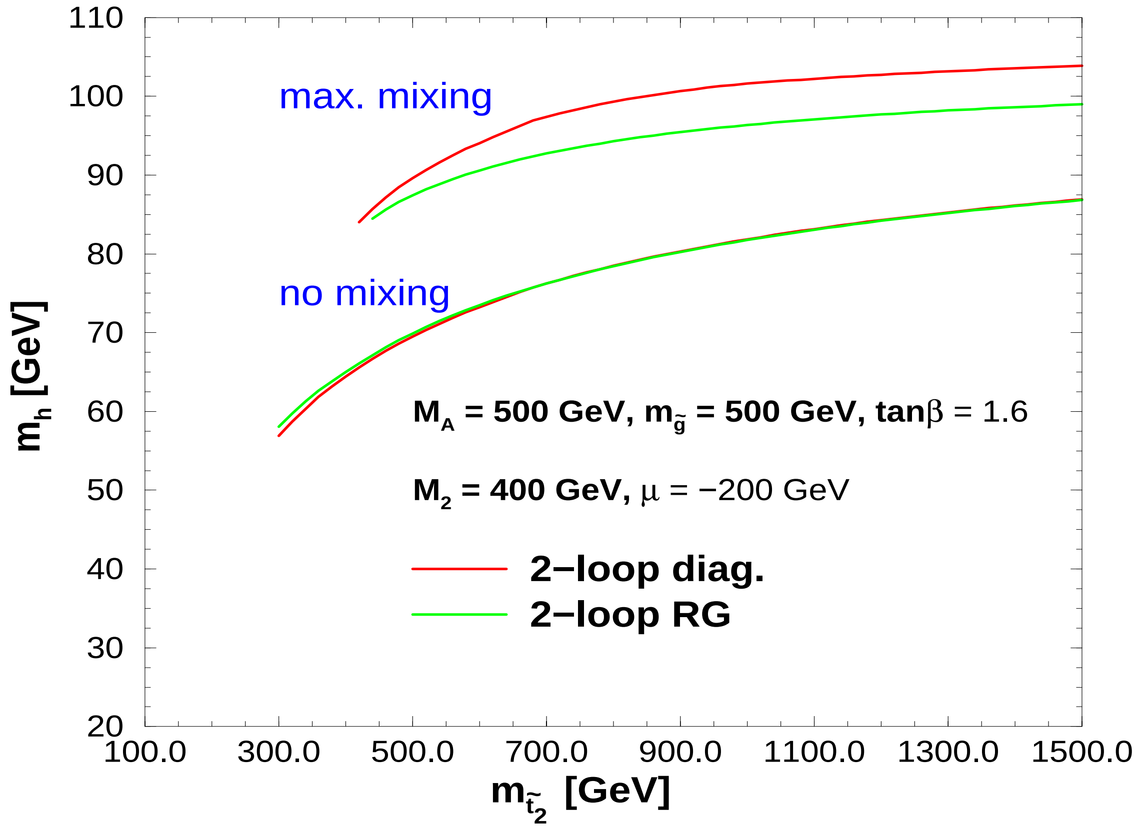
RG result: maximum for $|M_t^{LR}| \approx 2.4\, m_{\tilde{q}}$

$\leftrightarrow$ one-loop value

Problem: comparison in terms of unobservable parameters M_t^{LR}, $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$

From two-loop order on: different meaning in different renormalization schemes!

Physical parameters: $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$



Good agreement for no mixing in $\tilde{t}$ sector

$m_{\text{h}}^{\text{max}} \approx 5$ GeV higher than in RG result

Variation of $m_{\tilde{g}}$, M $\Rightarrow$ further deviations

Comparison of two-loop leading-log terms:
(from compact formula)

$$\Delta m^{2,(2),G_\mu\alpha_s}_{\mathrm{h,log}} = -\frac{G_\mu\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_{\mathrm{t}}^4\left[3\log^2\left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right)\right.$$
$$\left.+\,2\log\left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right) - 3\frac{\left(M_t^{LR}\right)^2}{M_{\mathrm{S}}^2}\log\left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right)\right]$$

Transformation into $\overline{\mathrm{MS}}$ scheme

⇒ Leading-log terms agree with RG result

[*M. Carena, H. Haber, S. Heinemeyer, W. Hollik, C. Wagner, G. W. '99*]

Non-logarithmic two-loop terms: **new**

$$\Delta m^{2,(2),G_\mu\alpha_s}_{\mathrm{h,non-log}} = -\frac{G_\mu\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_{\mathrm{t}}^4\left[4 - 6\frac{M_t^{LR}}{M_{\mathrm{S}}}\right.$$
$$\left. -\,8\frac{\left(M_t^{LR}\right)^2}{M_{\mathrm{S}}^2} + \frac{17}{12}\frac{\left(M_t^{LR}\right)^4}{M_{\mathrm{S}}^4}\right]$$

⇒ result is asymmetric in $\pm M_t^{LR}$

Increase in $m_{\mathrm{h}}^{\mathrm{max}}$ by ≈ 5 GeV

7. Conclusions

- Diagrammatic calculation of the leading $\mathcal{O}(\alpha\alpha_s)$ corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM

  Analytical result in terms of:
  $\tan\beta\,, M_\mathrm{A}, \mu, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \ldots$

- Simple approximation formula:
  Agrees with full result within ≈ 2 GeV for most of the parameter space

- Fortran programs:
  FeynHiggs, *FeynHiggsFast*

- Comparison with RG results:
  Agreement in leading logs
  m_h higher by up to ≈ 5 GeV

- Low-$\tan\beta$ scenario covered at LEP2
  But increased value for $m_\mathrm{h}^\mathrm{max}$
  $\Rightarrow$ accessible $\tan\beta$ region decreased

- Precise determination of m_h
  $\Rightarrow$ Sensitive test of the MSSM