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Production and decays of stops, sbottoms, and staus at an e^+e^- Linear Collider

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http://wwwhephy.oeaw.ac.at/p3w/theory/susy/

The Sfermions $\tilde{t}$, $\tilde{b}$, and $\tilde{\tau}$

- **YUKAWA couplings:**

$$h_t = \frac{m_t}{\sqrt{2} m_W \sin\beta}, \qquad h_{b,\tau} = \frac{m_{b,\tau}}{\sqrt{2} m_W \cos\beta}$$

$$\rightarrow \quad \frac{h_b}{h_t} = \frac{m_b}{m_t} \tan\beta \sim \frac{\tan\beta}{35}, \quad h_\tau^2 \sim \frac{1}{8} h_b^2$$

- **mixing angle:**

$$\tan 2\theta_{\tilde{f}} = \frac{2 a_f \, m_f}{m_{\tilde{f}_L}^2 - m_{\tilde{f}_R}^2}, \qquad f = t, b, \tau$$

with

$$a_t = (A_t - \mu \cot\beta) \qquad a_{b,\tau} = (A_{b,\tau} - \mu \tan\beta)$$

Therefore possible in stop– and sbottom/stau sector (for large $\tan\beta$):

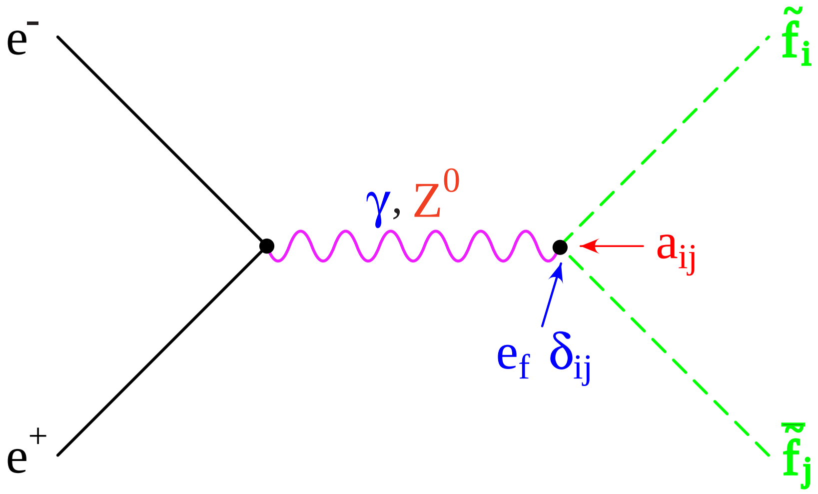
- large couplings to Higgs bosons
- possibly small 3rd gen. $M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}, M_{\tilde{L}}, M_{\tilde{E}}$ $\Leftarrow$ RGE's
- large off diagonal elements in mass matrix

$\rightarrow$ large L–R mixing and mass splitting,
light $\tilde{f}_1$ possible

$\rightarrow$ production cross section and decay branching ratios
can be very different
from 1st and 2nd generation sfermions!

Stop, Sbottom, and Stau Production

Tree Level



$$\tilde{f}_i^* \tilde{f}_j Z^0 \quad \ldots \quad \begin{cases} a_{11} = 4\,(I_f^{3L}\cos^2\theta_{\tilde{f}} - e_f \sin^2\theta_W) \\ a_{22} = 4\,(I_f^{3L}\sin^2\theta_{\tilde{f}} - e_f \sin^2\theta_W) \\ a_{12} = -2\,I_f^{3L}\sin 2\theta_{\tilde{f}} = a_{21} \end{cases}$$

$$\tilde{f}_i^* \tilde{f}_j \gamma^0 \quad \ldots \quad e_f \delta_{ij}$$

$$\sigma^{\rm tree} = \frac{c_{\tilde{f}}\,\pi\alpha^2\kappa_{ij}^3}{s^4}\left\{e_f^2\delta_{ij} - T_{\gamma Z}a_{ij}e_f\delta_{ij} + T_{ZZ}a_{ij}^2\right\}$$

unpol.

$c_{\tilde{f}} = \{1;\ 1/3\}$ for $\{\tilde{t}, \tilde{b};\ \tilde{\tau}\}$; $\kappa_{ij} = [(s - m_{\tilde{f}_i}^2 - m_{\tilde{f}_j}^2)^2 - 4m_{\tilde{f}_i}^2 m_{\tilde{f}_j}^2]^{1/2}$ ... kin. function;
e_f ... electric charge of fermion f; $s_W = \sin\theta_w$, $c_W = \cos\theta_w$;
a_{ij} ... $\tilde{f}_i\tilde{f}_j Z$ couplings; $v_e = 4\sin^2\theta_w - 1 \sim -0.08 \ll a_e = -1$ ... eeZ couplings.

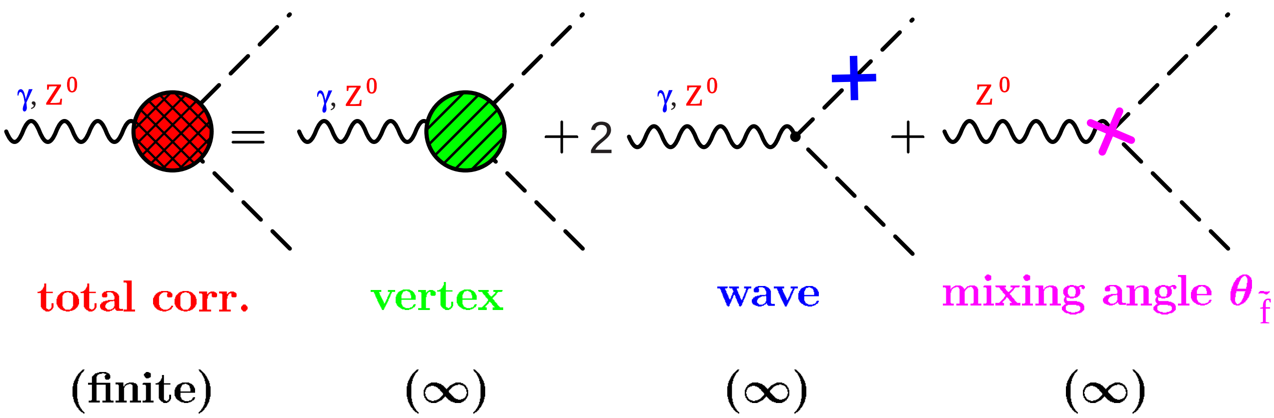
$$T_{\gamma Z} = \frac{v_e}{8c_W^2 s_W^2} D_{\gamma Z}, \qquad T_{ZZ} = \frac{(a_e^2 + v_e^2)}{256 s_W^4 c_W^4} D_{ZZ}, \quad \text{with}$$

$$D_{\gamma Z} = \frac{s\,(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}, \quad D_{ZZ} = \frac{s^2}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}.$$

One–loop corrections

Three parts:

- conventional QCD corrections[1]
  — gluon exchange and radiation
- corrections due to gluino and squark exchange[2]
- corrections due to Yukawa interactions of sfermions[3]
  chargino, neutralino, charged and neutral Higgs exchange, $\mathcal{O}(h_f^2)$



$$\Delta a_{ij} = \delta a_{ij}^{(\text{vertex})} + \delta a_{ij}^{(\text{wave})} + \delta a_{ij}^{(\tilde\theta)}$$

$$(\Delta e_{\mathrm{f}})_{ij} = (\delta e_{\mathrm{f}})_{ij}^{(\text{vertex})} + (\delta e_{\mathrm{f}})_{ij}^{(\text{wave})}$$

- **We used the on–shell renormalization scheme and the SUSY conserving dimensional reduction regularisation scheme (DimRed)**
  (here equivalent to DimReg)

- **Calculations are done in the unitary gauge** (G^+ and G^0 treated in $\xi = 1$ gauge)

- $\sigma^{\text{corr}} = \sigma^{\text{tree}}$ with the substitutions
  $e_f \delta_{ij} \to e_f \delta_{ij} + (\Delta e_f)_{ij}$, $a_{ij} \to a_{ij} + \Delta a_{ij}$

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[1] Drees, Hikasa; Beenakker, Höpker, Zerwas
[2] H. E., Bartl, Majerotto; Arhrib, Capdequi–Peyranere, Djouadi
[3] H. E., Kraml, Majerotto, hep-ph/9903413

Parameter point:

Gaugino scenario

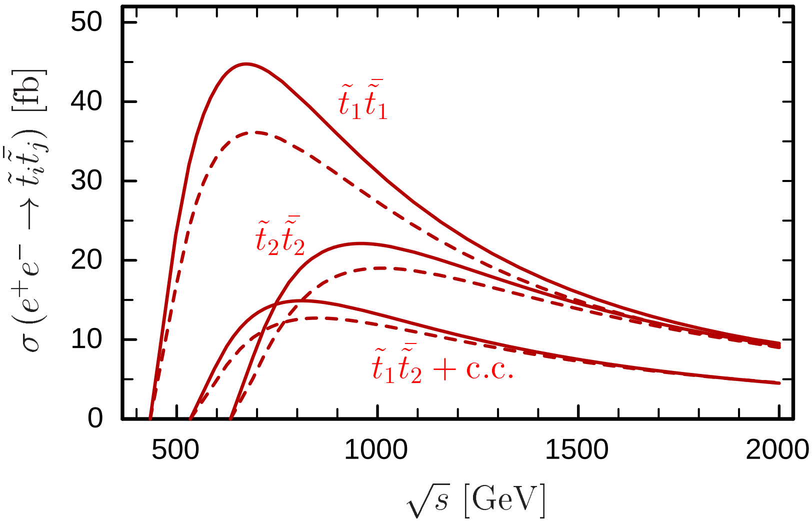
(all masses in GeV)

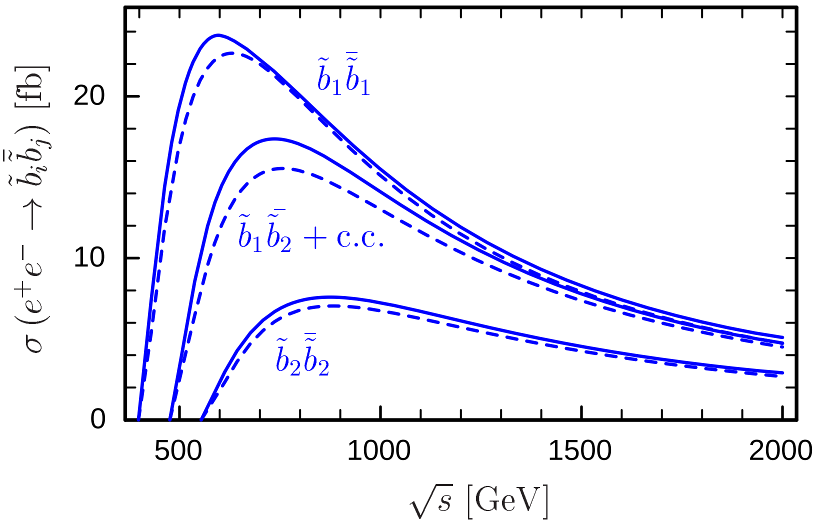
M	$=$	$200,$	μ	$=$	$1000,$	$\tan\beta$	$=$	$4,$	m_A	$=$	$300,$
$M_{\tilde{Q}}$	$=$	$225,$	$M_{\tilde{U}}$	$=$	$200,$	$M_{\tilde{D}}$	$=$	$250,$	$A_{t,b}$	$=$	$400,$
$M_{\tilde{L}}$	$=$	$280,$				$M_{\tilde{E}}$	$=$	$250,$	A_τ	$=$	$100.$

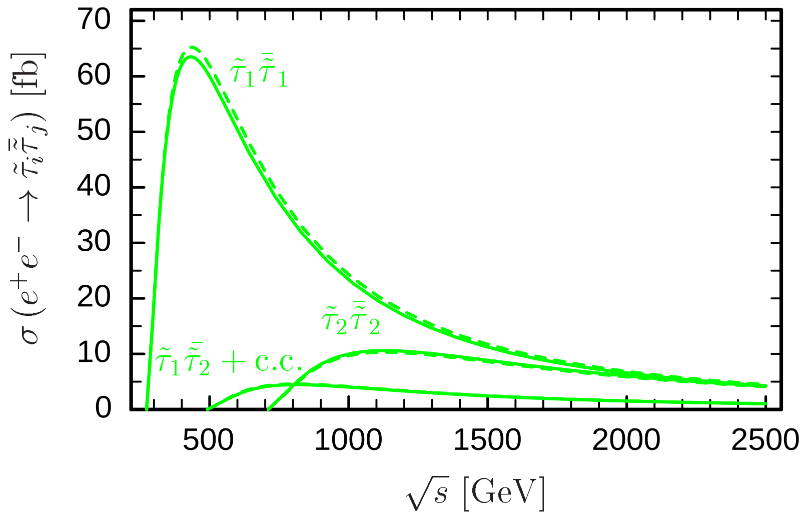
$m_{\tilde{t}_1}$	$=$	$218,$	$m_{\tilde{t}_2}$	$=$	$317,$	$\cos\theta_{\tilde{t}}$	$=$	$-0.64,$			
$m_{\tilde{b}_1}$	$=$	$200,$	$m_{\tilde{b}_2}$	$=$	$278,$	$\cos\theta_{\tilde{b}}$	$=$	$0.79,$			
$m_{\tilde{\tau}_1}$	$=$	$137,$	$m_{\tilde{\tau}_2}$	$=$	$355,$	$\cos\theta_{\tilde{\tau}}$	$=$	$0.65,$	$\tan\beta = 30$		
$m_{\tilde{\chi}_1^\pm}$	$=$	$196,$	$m_{\tilde{\chi}_2^\pm}$	$=$	$1007,$	$m_{\tilde{g}}$	$=$	$559,$			
$m_{\tilde{\chi}_1^0}$	$=$	$100,$	$m_{\tilde{\chi}_2^0}$	$=$	$196,$	$m_{\tilde{\chi}_3^0}$	$=$	$1002,$	$m_{\tilde{\chi}_4^0}$	$=$	$1007,$
m_{h^0}	$=$	$93,$	m_{H^0}	$=$	$305,$	m_{H^+}	$=$	$298,$	$\sin\alpha$	$=$	$-0.32.$

Total cross sections

including SUSY–QCD and Yukawa coupling corrections



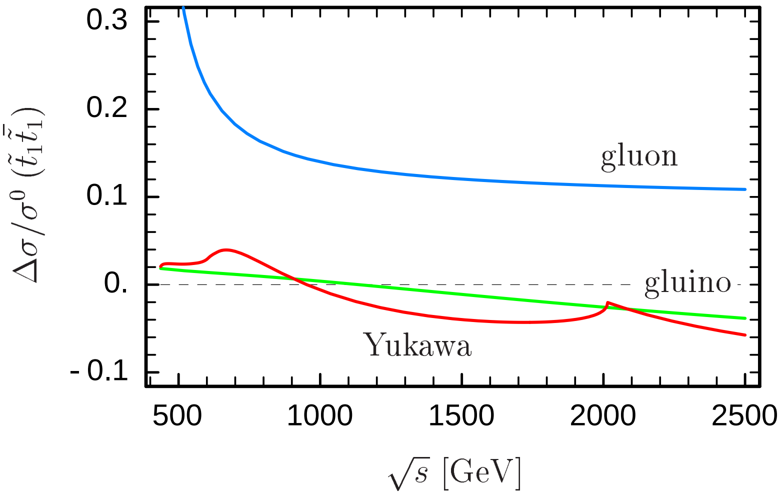




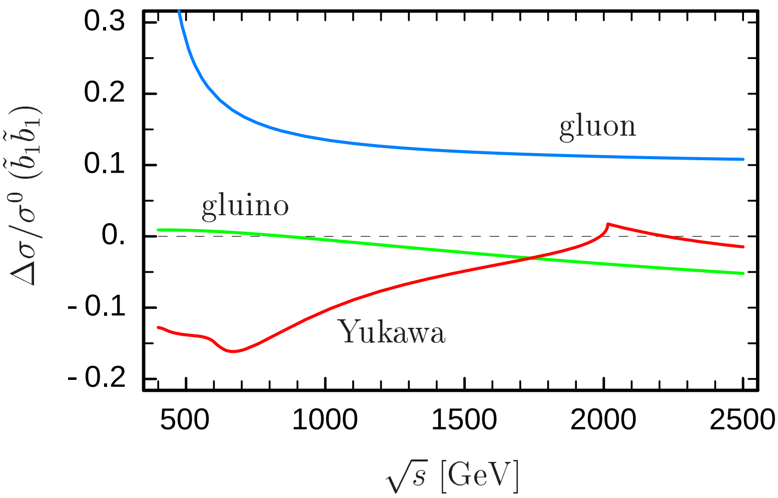
Radiative corrections

relative to tree–level Xsection

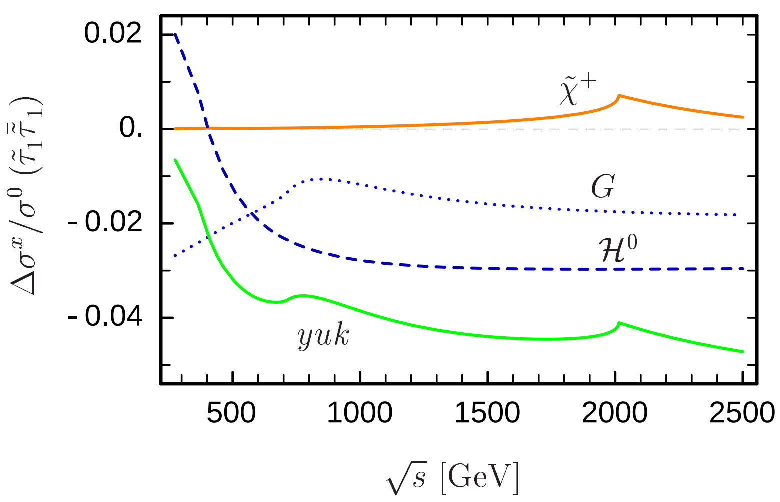
$e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ $\tan\beta = 4$



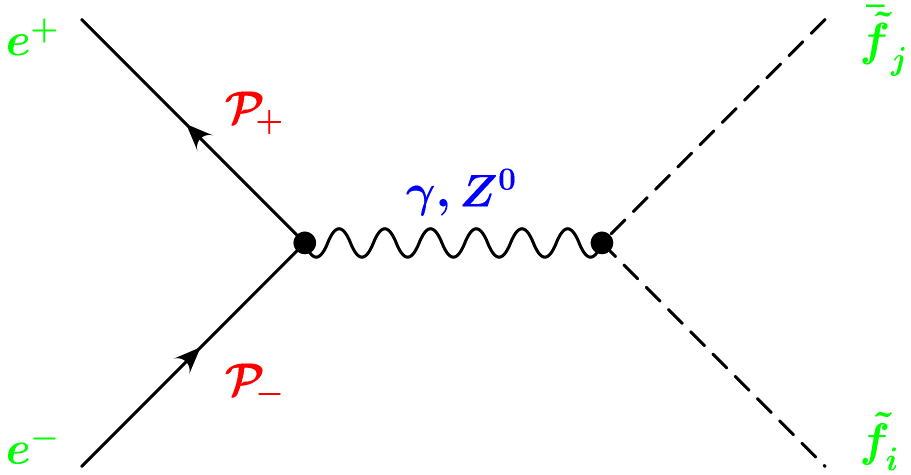
$e^+e^- \to \tilde{b}_1\bar{\tilde{b}}_1$ $\tan\beta = 4$



$e^+e^- \to \tilde{\tau}_1\bar{\tilde{\tau}}_1$ $\tan\beta = 30$



Polarization of $e^{\pm}$ beams



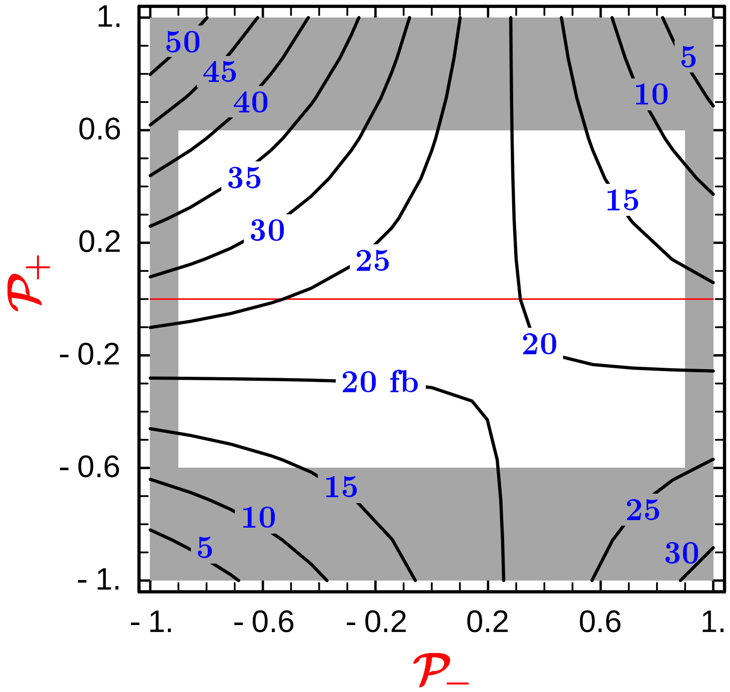
$\mathcal{P}_-$ is the degree of polarization of the e^- beam and $\mathcal{P}_+$ that of the e^+ beam. $\mathcal{P}_\pm \in [-1, 1]$ with

$$\mathcal{P}_\pm = \begin{cases} -1 & \text{if } e^\pm \text{ is left–polarized,} \\ 0 & \text{if } e^\pm \text{ is unpolarized,} \\ +1 & \text{if } e^\pm \text{ is right–polarized.} \end{cases}$$

$$\begin{aligned} &\sigma^{\rm tree}(e^+e^- \to \tilde{f}_i \bar{\tilde{f}}_j) = \\ &\frac{c_{\tilde{f}}\, \pi \alpha^2 \kappa_{ij}^3}{s^4} \Big\{ e_f^2\, \delta_{ij}\, (1 - \mathcal{P}_-\mathcal{P}_+) \\ &\qquad - \frac{e_f a_{ij} \delta_{ij}}{8\, \mathrm{s}_W^2 \mathrm{c}_W^2} \big[v_e\, (1 - \mathcal{P}_-\mathcal{P}_+) - a_e\, (\mathcal{P}_- - \mathcal{P}_+) \big]\, D_{\gamma Z} \\ &\qquad + \frac{a_{ij}^2}{256\, \mathrm{s}_W^4 \mathrm{c}_W^4} \big[(v_e^2 + a_e^2)\, (1 - \mathcal{P}_-\mathcal{P}_+) \\ &\qquad\qquad - 2\, v_e a_e\, (\mathcal{P}_- - \mathcal{P}_+) \big]\, D_{ZZ} \Big\} \end{aligned}$$

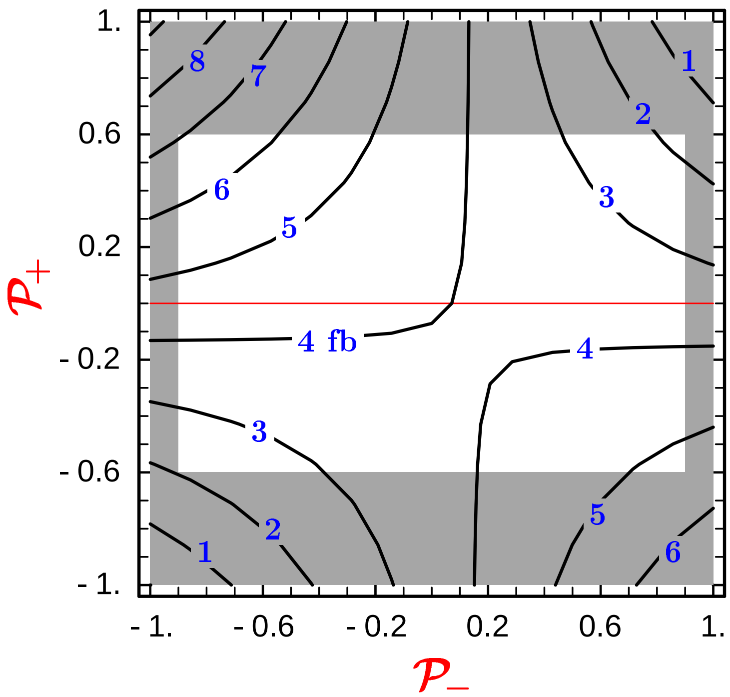
[hep-ph/9903257]

$e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$



$\sqrt{s} = 500$ GeV

$e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_2$



$\sqrt{s} = 800$ GeV

$m_{\tilde{t}_1} = 220$ GeV

$m_{\tilde{t}_2} = 415$ GeV

$\cos\theta_{\tilde{t}} = -0.66$

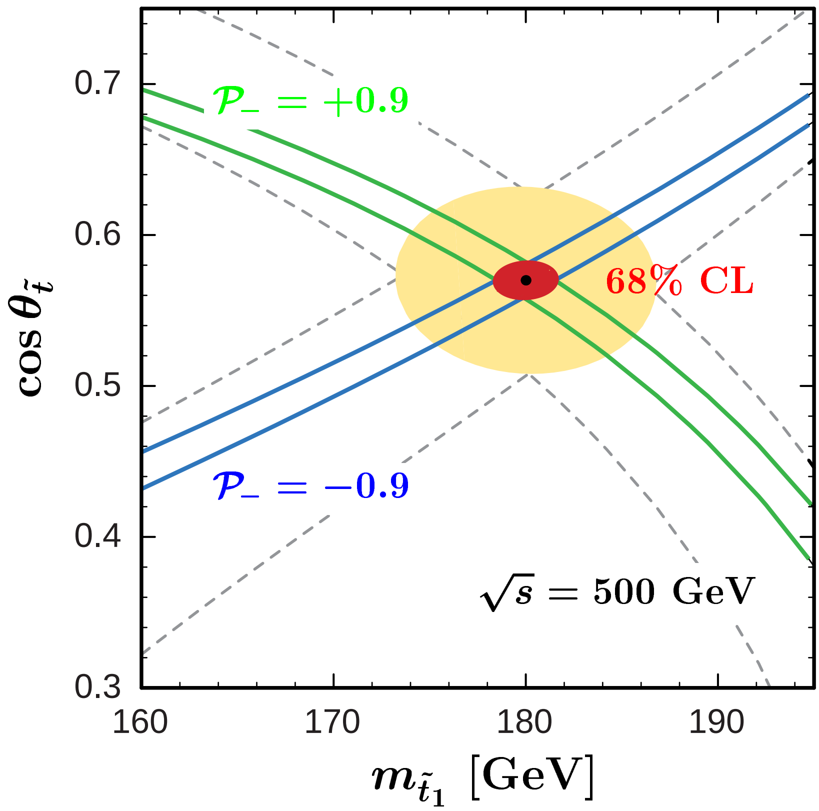
$\mathcal{P}_-$ Polarization of e^- beam

$\mathcal{P}_+$ Polarization of e^+ beam

Determination of MSSM parameters with polarized e^- beams

$$\int \mathcal{L} = 300\ \text{fb}^{-1}$$

$$m_{\tilde{t}_1} = 180\ \text{GeV}, \quad \cos\theta_{\tilde{t}} = 0.57$$



we ESTIMATE errors

$$\mathcal{P}_- = -0.9 \quad \ldots \quad \sigma\,(\tilde{t}_1\bar{\tilde{t}}_1) = 61.2 \pm 1.5\ \text{fb}$$
$$\mathcal{P}_- = +0.9 \quad \ldots \quad \sigma\,(\tilde{t}_1\bar{\tilde{t}}_1) = 57.1 \pm 1.4\ \text{fb}$$

SUSY–QCD corrections taken into account

$$\Rightarrow \quad m_{\tilde{t}_1} = 180 \pm 1.65\ \text{GeV}, \quad \cos\theta_{\tilde{t}} = 0.57 \pm 0.012$$

$\int \mathcal{L} = 10\ \text{fb}^{-1}$:[4] 7 GeV, 0.06

[4] Bartl et al., Z. Phys.

Parameter point:

(all masses in GeV)

$$m_{\tilde{t}_1} = 220.2\,,\quad m_{\tilde{t}_2} = 414.7\,,\quad \cos\theta_{\tilde{t}} = -0.6606\,,$$
$$m_{\tilde{b}_1} = 296.9\,,\quad m_{\tilde{b}_2} = 345.5\,,\quad \cos\theta_{\tilde{b}} = 0.827\,,$$
$$m_{A^0} = 300\,,\qquad \mu = 800\,,\qquad \tan\beta = 4\,,$$
$$M = 200\,.$$

$\Rightarrow$

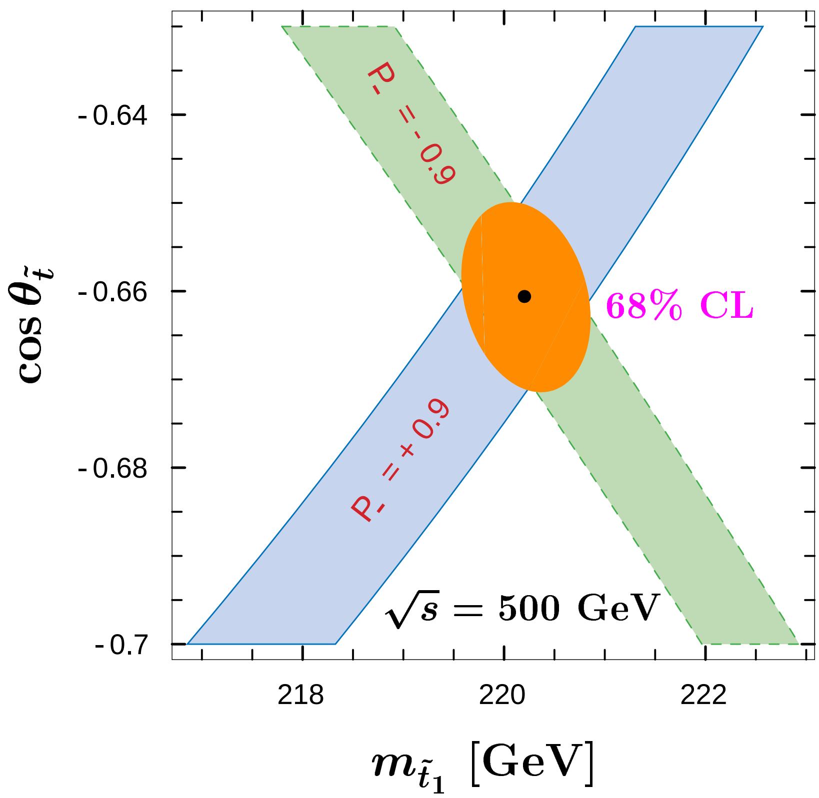
$$m_{h^0} = 104.7\,,\quad m_{H^0} = 304.2\,,\quad m_{H^+} = 306.4\,,$$
$$m_{\tilde{\chi}^+_1} = 194.0\,,\quad m_{\tilde{\chi}^+_2} = 809.4\,,$$
$$m_{\tilde{\chi}^0_1} = 99.2\,,\quad m_{\tilde{\chi}^0_2} = 194.1\,,\quad m_{\tilde{\chi}^0_3} = 802.3\,,\quad m_{\tilde{\chi}^0_4} = 809.7\,,$$
$$m_{\tilde{g}} = 558.8\,.$$

Decay of	$\tilde{t}_1 \to b\,\tilde{\chi}^+_1$	...	100 %
Decay of	$\tilde{t}_2 \to t\,\tilde{\chi}^0_1$	...	7.0 %
	$\to t\,\tilde{\chi}^0_2$	...	7.2 %
	$\to b\,\tilde{\chi}^+_1$	...	33.5 %
	$\to \tilde{t}_1\,Z^0$	...	32.7 %
	$\to \tilde{b}_1\,W^+$	...	19.0 %
	$\to \tilde{t}_1\,h^0$	...	0.6 %

Determination of MSSM parameters with polarized e^- beam

$\int \mathcal{L} = 500\ \mathrm{fb}^{-1}$

$m_{\tilde{t}_1} = 220.2$ GeV, $\cos\theta_{\tilde{t}} = -0.6606$



we ESTIMATE errors

$\mathcal{P}_- = -0.9$ ... $\sigma(\tilde{t}_1\bar{\tilde{t}}_1) = 27.2 \pm 0.6$ fb

$\mathcal{P}_- = +0.9$ ... $\sigma(\tilde{t}_1\bar{\tilde{t}}_1) = 16.5 \pm 0.5$ fb

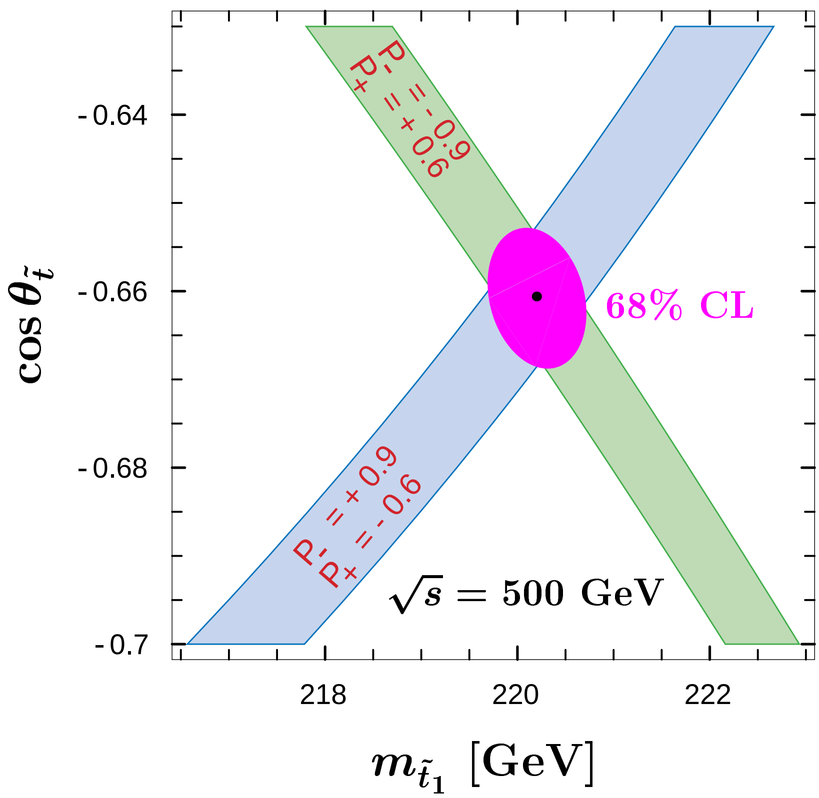
SUSY–QCD and Yukawa coupl. corr. taken into account

$\Rightarrow$ $m_{\tilde{t}_1} = 220.2 \pm 0.64$ GeV, $\cos\theta_{\tilde{t}} = -0.6606 \pm 0.0108$

Determination of MSSM parameters with polarized e^- and e^+ beams

$\int \mathcal{L} = 500 \text{ fb}^{-1}$

$m_{\tilde{t}_1} = 220.2$ GeV, $\quad \cos\theta_{\tilde{t}} = -0.6606$



we ESTIMATE errors

$(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, +0.6) \quad \ldots \quad \sigma\,(\tilde{t}_1\bar{\tilde{t}}_1) = 42.6 \pm 0.8$ fb

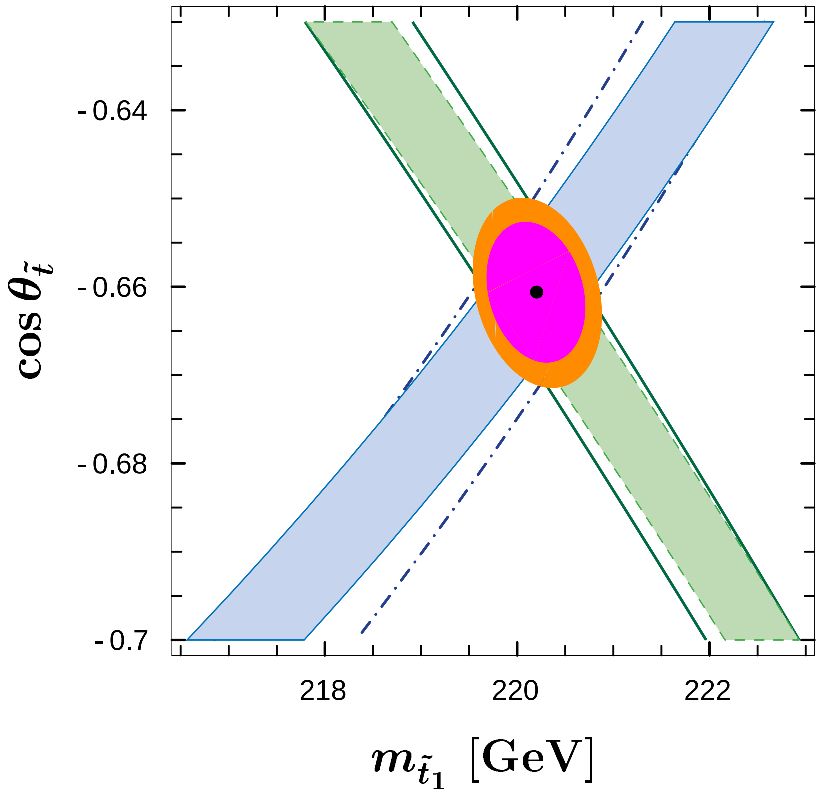
$(\mathcal{P}_-, \mathcal{P}_+) = (+0.9, -0.6) \quad \ldots \quad \sigma\,(\tilde{t}_1\bar{\tilde{t}}_1) = 24.8 \pm 0.6$ fb

SUSY–QCD and Yukawa coupl. corr. taken into account

$\Rightarrow \quad m_{\tilde{t}_1} = 220.2 \pm 0.52$ GeV, $\quad \cos\theta_{\tilde{t}} = -0.6606 \pm 0.0080$

Comparison of results

pol. e^- beam $\leftrightarrow$ pol. e^- and e^+ beams



$$\left.\begin{array}{rl} \Delta m_{\tilde{t}_1}: & 0.64\,\text{GeV} \Rightarrow 0.52\,\text{GeV} \\ \Delta\cos\theta_{\tilde{t}}: & 0.0108 \Rightarrow 0.0080 \end{array}\right\} \sim 25\%\ \text{red.}$$

$\sqrt{s} = 800$ GeV:

$$\mathcal{P}_- = -0.9 \quad \ldots \quad \sigma\,(\tilde{t}_1\bar{\tilde{t}}_2) = 4.55 \pm 0.19\,\text{fb}$$

$$(\mathcal{P}_-, \mathcal{P}_+) = (-0.9, +0.6) \quad \ldots \quad \sigma\,(\tilde{t}_1\bar{\tilde{t}}_2) = 7.07 \pm 0.24\,\text{fb}$$

$$\Rightarrow m_{\tilde{t}_2} = 415 \pm \begin{cases} 5.0\ \text{GeV} \\ 4.1\ \text{GeV} \end{cases}$$

Precision of MSSM parameters in stop sector

assume

$$\mu = 800 \pm 80\,\text{GeV}$$

$$\tan\beta = 4 \pm 0.4\,\text{GeV}$$

then

	e^- pol.		e^- & e^+pol.	
$m_{\tilde{Q}} = 300.2$	± 4.9	|	± 3.9	GeV
$m_{\tilde{U}} = 270.2$	± 4.7	|	± 3.7	GeV
$A_t = 550.7$	± 30.7	|	± 29.9	GeV

If $\Delta\,\mu \to 0$

$$\Delta\, A_t = \pm 11.9 \quad | \quad \pm 9.7 \quad \text{GeV}$$

If we assume $\Delta m_{\tilde{t}_2} = 1$ GeV

e^-&e^+pol.:

$$\Delta\, m_{\tilde{Q}} = 2.3\,\text{GeV}, \quad \Delta\, m_{\tilde{U}} = 2.5\,\text{GeV}, \quad \Delta\, A_t = 28.4\,\text{GeV}$$

Decays of Stops, Sbottoms, and Staus

into FERMIONS:

$$\tilde{t}_i \to b + \tilde{\chi}^+_j \qquad \tilde{t}_i \to t + \tilde{\chi}^0_k \qquad \tilde{t}_i \to t + \tilde{g}$$
$$\tilde{b}_i \to t + \tilde{\chi}^-_j \qquad \tilde{b}_i \to b + \tilde{\chi}^0_k \qquad \tilde{b}_i \to b + \tilde{g}$$
$$\tilde{\tau}_i \to \nu_\tau + \tilde{\chi}^-_j \qquad \tilde{\tau}_i \to \tau + \tilde{\chi}^0_k$$

into BOSONS:

$$\tilde{t}_i \to \tilde{b}_j + W^+,\ H^+ \qquad \tilde{t}_2 \to \tilde{t}_1 + Z^0,\ h^0, H^0,\ A^0$$
$$\tilde{b}_i \to \tilde{t}_j + W^-,\ H^- \qquad \tilde{b}_2 \to \tilde{b}_1 + Z^0,\ h^0, H^0,\ A^0$$
$$\tilde{\tau}_i \to \tilde{\nu}_\tau + W^-,\ H^- \qquad \tilde{\tau}_2 \to \tilde{\tau}_1 + Z^0,\ h^0, H^0,\ A^0$$

$\tilde{t}, \tilde{b}$: Gluino mode is important if kinematically allowed!

Bosonic decays of $\tilde{f}_2 \leftarrow$ sfermion mass splitting!

Higher order decays of e.g. $\tilde{t}_1$:[5]

- $m_{\tilde{t}_1} < m_b + m_{\tilde{\chi}^+_1}$:

$\tilde{t}_1 \to c + \tilde{\chi}^0_k$
$\tilde{t}_1 \to b + W^+ + \tilde{\chi}^0_k$
$\tilde{t}_1 \to b + H^+ + \tilde{\chi}^0_k$

- $m_{\tilde{\ell},\tilde{\nu}} + m_b < m_{\tilde{t}_1}$:

$\tilde{t}_1 \to b + \ell + \tilde{\nu}_\ell, b + \nu_\ell + \tilde{\ell}$

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[5] W. Porod, PhD-thesis, hep-ph/9804208, and hep-ph/9812230

SUSY–QCD corrections to squark decay widths

- $\tilde{q}_i \to q\,\tilde{\chi}^0_k,\; q'\,\tilde{\chi}^\pm_j$

  Kraml, H. E., Bartl, Majerotto, Porod, Phys. Lett B 386 (1996) 175
  Djouadi, Hollik, Jünger, Phys. Rev. D 55 (1997) 6975
  Beenakker, Höpker, Plehn, Zerwas, Z. Phys. C 75 (1997) 349

- $\tilde{q}_2 \to \tilde{q}_1\, Z^0,\; \tilde{q}_i \to \tilde{q}'_j\, W^\pm$

  Bartl, H. E., Hidaka, Kraml, Majerotto, Porod, Yamada, Phys. Lett B 419 (1998) 243

- $\tilde{q}_2 \to \tilde{q}_1 + (h^0, H^0, A^0),\; \tilde{q}_i \to \tilde{q}'_j\, H^\pm$

  Arhrib, Djouadi, Hollik, Jünger, Phys. Rev. D 57 (1998) 5860
  Bartl, H. E., Hidaka, Kraml, Majerotto, Porod, Yamada, hep-ph/9806299, to be publ. in PRD

- $\tilde{q}_i \to q\,\tilde{g}$

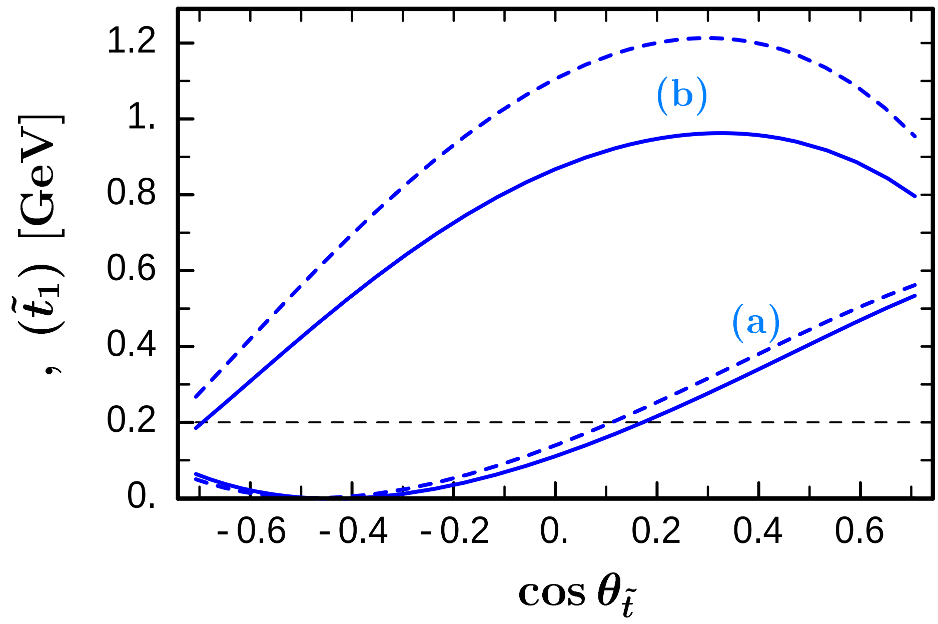
  Beenakker, Höpker, Plehn, Zerwas, Z. Phys. C 75 (1997) 349

can change the tree–level decay widths by a few ten percent!

Decay width of $\tilde{t}_1 \to b\,\tilde{\chi}_1^+$

$m_{\tilde{t}_1} = 200\,\mathrm{GeV}, \quad m_{\tilde{\chi}_1^\pm} = 133\,\mathrm{GeV},$

$m_{\tilde{t}_2} = 490\,\mathrm{GeV}, \quad \tan\beta = 3$



(a) $M = 160$ GeV, $\mu = 300$ GeV

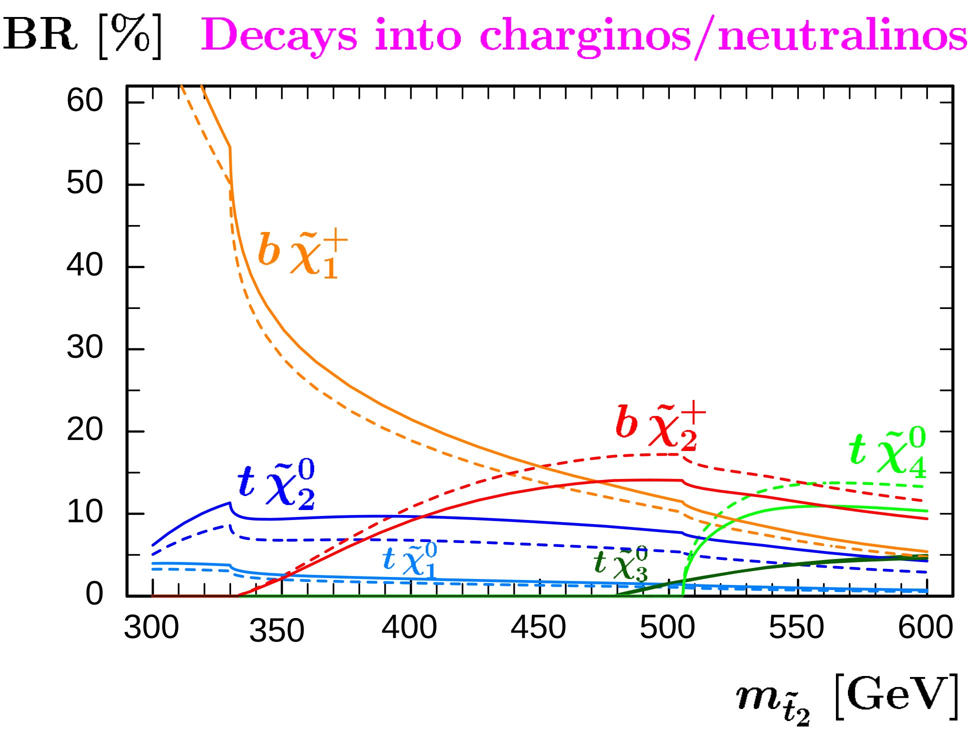
(b) $M = 300$ GeV, $\mu = 160$ GeV

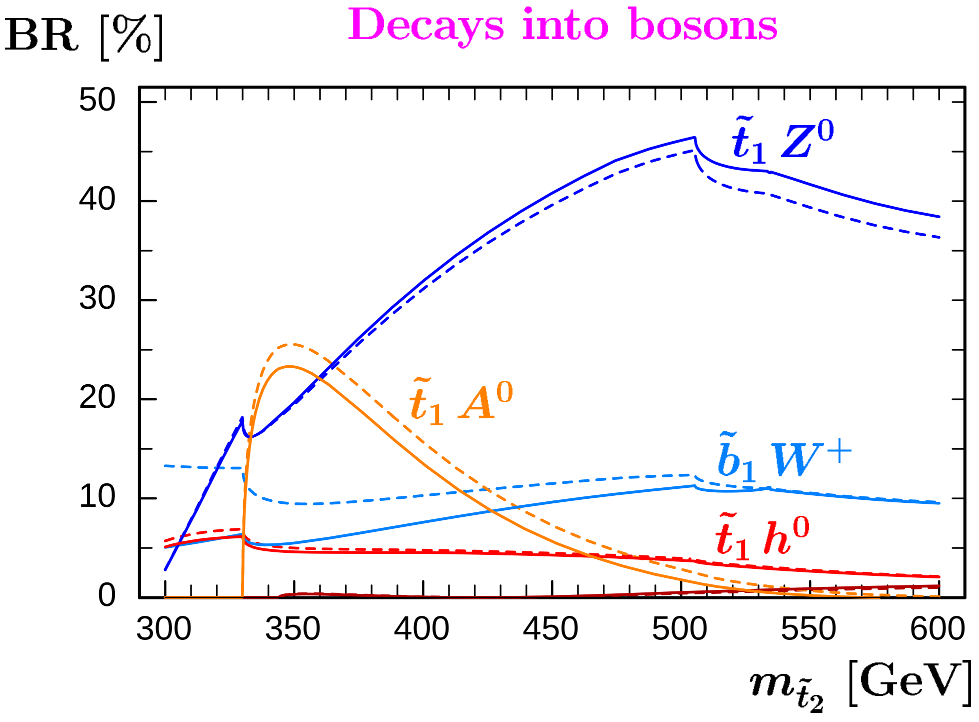
dashed lines ... tree level

full lines ... SUSY–QCD corrected

BR's of $\tilde{t}_2$ decays as a function of $m_{\tilde{t}_2}$

$m_{\tilde{t}_1} = 200$ GeV , $\cos\theta_{\tilde{t}} = 0.6$
$M = 120$ GeV , $m_{A^0} = 130$ GeV , $\tan\beta = 3$
$\mu = 300$ GeV , $A_t = A_b$, $M_{\tilde{D}} = 1.1\, M_{\tilde{Q}}$





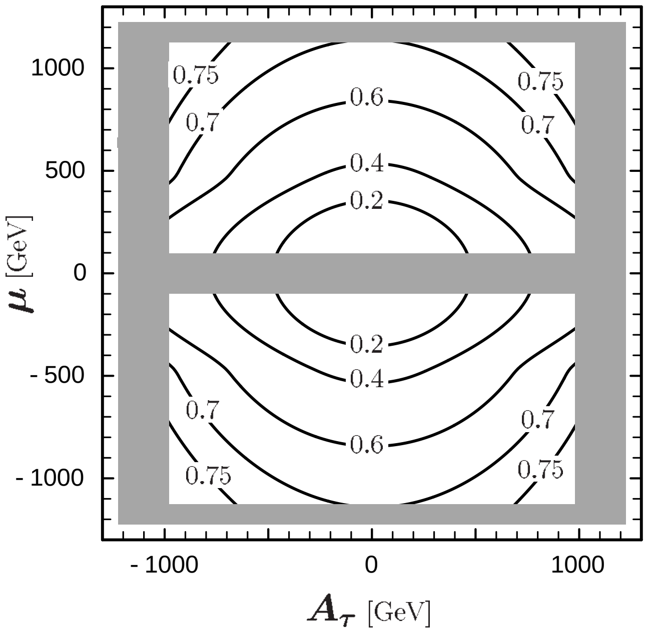
$\tilde{\tau}_2$ decays

bosonic versus fermionic modes[6]

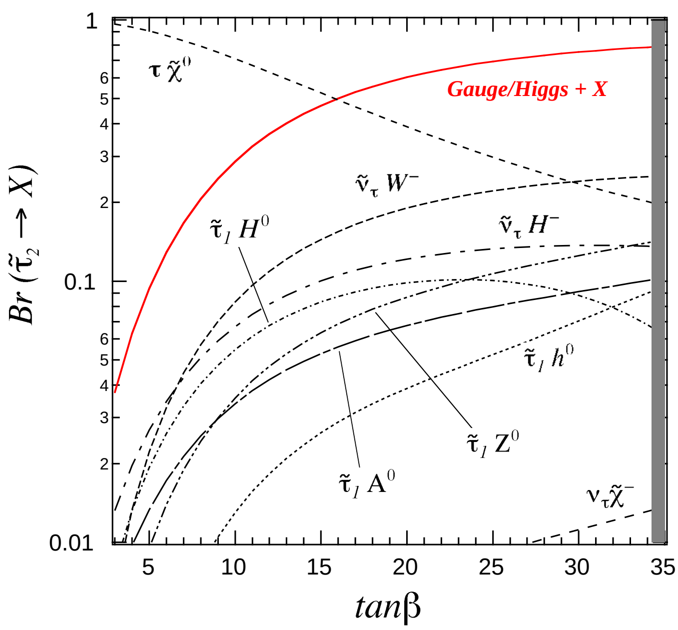
$m_{\tilde{\tau}_1} = 250$ GeV
$m_{\tilde{\tau}_2} = 500$ GeV
case $m_{\tilde{\tau}_L} < m_{\tilde{\tau}_R}$

$M = 300$ GeV, $m_{A^0} = 150$ GeV

$\sum \mathrm{Br}(\tilde{\tau}_2 \to \text{bosons})$



$\tan\beta = 30$



$A_\tau = 800$ GeV
$\mu = 1000$ GeV

[6]Bartl et al., hep-ph/9904417

Conclusions

- **$e^+e^- \to$ Stops, Sbottoms, and Staus, LC energies: cross section is about 1 to 200 fb**

- **Beam (e^-) polarization necessary for a precise determination of the basic parameters of sfermions**

- **e^+ polarization improves the precision of parameter determination**

- **Measurement of Xsection is in $\mathcal{O}(\%)$**
  $\Rightarrow$ SUSY–QCD and Yukawa coupling corr.
  must be taken into account!

- **$\tilde{t}_1$ — $\tilde{b}_1$: different decay patterns due to large m_t**

- **$\tilde{f}_2$ has a wide spectrum of possible decay modes**

- **bosonic decay modes of $\tilde{f}_2$ can have large BR's**

- **SUSY–QCD corrections can be sizeable**
