A DIRECT RECONSTRUCTION OF THE COMPLEX PARAMETER CASE IN THE GAUGINO SECTOR

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Physics and experiments at future linear e^+e^- Colliders

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1. Motivations

unconstrained MSSM = large nb of (soft-breaking) parameters

Moreover, some may be complex

Constraints on φ exist (EDM, FCNC,..) | Ibrahim, Nath '97

but "large" phases not excluded. | Brhlik, Good, Kane '98

→ either → more th. assumptions (minimal SUGRA, ...)

↘ more "pragmatic" strategy maybe also worthwhile

if SUSY discovered, next (LC?) task:

assuming masses, cross-section measured for (some) superpartners

- what impact on values (bounds) of $\mathcal{L}$ soft-breaking?
- Are non-trivial phases mandatory?
- Are data consistent with relationships as implied in MSSM?

→ Conventional strategy: • systematic scan of $\mathcal{L}_{MSSM}$
• Fit (once data) and extract $\mathcal{L}_{MSSM}$

≃ OK for mSUGRA, but eventually obscure..

OUR AIM: a more DIRECT (= analytic) INVERSION of the Lagrangian-to-physical relationships
"DE-DIAGONALIZATION"

NB: never replaces specific exp. analysis procedure
BUT complementary SIMULATION TOOL:

ADVANTAGES:
- simple (relatively) analytic expressions → Fast algorithms
- CLEARER identification of possible ambiguities (multifold solutions)
- OFTEN GIVES a better "insight" on Lagrangian-to-physical parameter interplay

(e.g. NON-TRIVIAL chargino/neutralino parameter correlations explicit

2. Relevant parameters in the ino SECTOR

charginos: $M_C = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & |\mu| e^{i\phi_\mu} \end{pmatrix} \to m_{\chi^+_{1,2}}$; $\underbrace{\phi_L, \phi_R}_{\downarrow}$ mixing

e.g $Z \chi^+_i \chi^-_j$ Coupling

Neutralinos:

$$M_N = \begin{pmatrix} |M_1| e^{i\phi_{M_1}} & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M_2 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -|\mu| e^{i\phi_\mu} \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -|\mu| e^{i\phi_\mu} & 0 \end{pmatrix}$$

$\to m_{N_{1,2,3,4}}$; Z_{ij} (diag. matrix el^{t}s) $\to$ e.g $Z \chi^0_i \chi^0_j$ Couplings

physically relevant phases: | M_2 real ≥ 0 convention choice;
| ϕ_μ, ϕ_{M_1} (Consistent with e.g. EDM analysis)

(i.e. other possible phases absorbed by parameters (fields) redefinition freedom)

Basic algorithms:

* real case ($\phi_{\mu, M_1} \equiv 0$) { "S_1": input { $m_{\chi^+_1}, m_{\chi^+_2}$; $m_{N_{any\,i}}$; (+ $\tan\beta$) } $\to \mu, M_2$; M_1 + all others m_{N_j}
"S_2": input { $m_{\chi^+_1}$; m_{N_i}, m_{N_j} ; $\tan\beta$ }

* Complex case input: { $m_{\chi^+_1}, m_{\chi^+_2}$, ϕ_L } (e.g $Z\chi^+_1\chi^+_2$)
↳ M_2, $|\mu|$, ϕ_μ
+ { M_{N_i}, M_{N_j} } (2 among any)
↳ $|M_1|$, ϕ_{M_1} + M_{N_i} (others)
+ Z_{ij}

+ Calculate $\sigma(e^+e^- \to \chi^0_i \chi^0_j)$ in Complex case

More precisely, in S_1 (real case)

$$\begin{pmatrix}\mu^2\\ M_2^2\end{pmatrix} = \frac{1}{2}\left[m_{\chi_1^+}^2 + m_{\chi_2^+}^2 - 2m_W^2 \mp \sqrt{(m_{\chi_1^+}^2 + m_{\chi_2^+}^2 - 2m_W^2)^2 - 4(m_W^2 \sin 2\beta \pm m_{\chi_1^+} m_{\chi_2^+})^2}\right]$$

$|\mu| < M_2$ ($m_{\chi_1^+}$ higgsino-like)

($|\mu| > M_2$ case trivially obtained: $|\mu| \leftrightarrow M_2$)

Further ambiguity when both $\pm$ expressions $\geqslant 0$

(depends crucially on $m_{\chi_2^+} - m_{\chi_1^+}$ and $\tan\beta$)

$$M_2 \mu = m_W^2 \sin 2\beta \pm m_{\chi_1^+} m_{\chi_2^+} \qquad \text{(fixes } \mu \text{ sign)}$$

NB: $M_2 \geqslant 0$; μ and M_1 have arbitrary signs

Neutralinos: trick is to use the 4 rotation-invariants:

1) $\mathrm{tr}\, M_N$: $\sum_1^4 M_{N_i} = M_1 + M_2$

2) $M_{N_1} M_{N_2}$ + permutations $= M_1 M_2 - \mu^2 - m_Z^2$

3) $M_{N_1} M_{N_2} M_{N_3}$ + permutations $= \mu\, m_Z^2 \sin 2\beta - (c_W^2 m_Z^2 + \mu^2) M_1 - (s_W^2 m_Z^2 + \mu^2) M_2$

4) $\det M_N$: $M_{N_1} M_{N_2} M_{N_3} M_{N_4} = \mu\, m_Z^2 (c_W^2 M_1 + s_W^2 M_2) \sin 2\beta - \mu^2 M_1 M_2$

$\to$ Rq: linear in M_1! (once μ, M_2 known from charginos)

$$M_1 = -\frac{P^2 + P(\mu^2 + m_Z^2 + M_2 S - S^2) + \mu M_2 m_Z^2 s_W^2 \sin 2\beta}{P(S - M_2) + \mu(c_W^2 m_Z^2 \sin 2\beta - \mu M_2)}$$

$S \equiv \underline{M_{N_i}} + M_{N_j}$ (any i, j's)

$P \equiv \underline{M_{N_i}} M_{N_j}$

M_{N_j} : given by cubic equations

(but ! unique solution from cross-consistency)

$M_{\chi_1^\pm} = 400$ GeV

$M_{N_2} = 50$ GeV

$\tan\beta = 2$

excluded by M_2, μ real

JLK + Moultaka
hep-ph/9807233
PRD 59 (99)

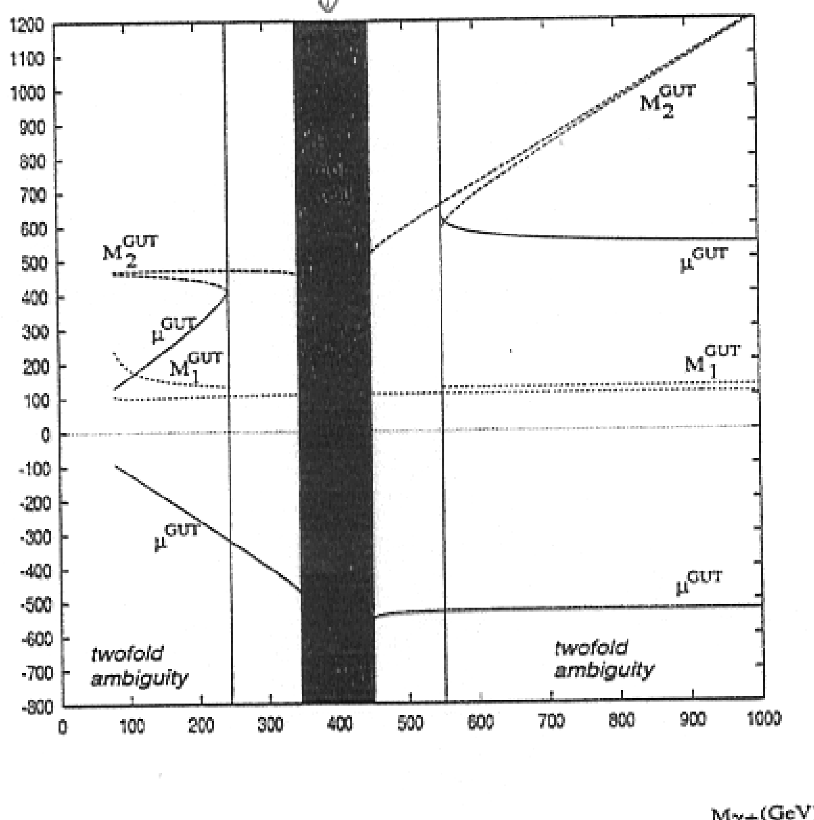


Figure 4: Same captions as for fig. 2 but with $\boxed{\mu,\ M_2 \text{ and } M_1 \text{ evolved up to GUT scale.}}$

Note: • $|\mu| \simeq M_{\chi_2^+}$ for $M_{\chi_2^+} \gg M_{\chi_1^+}$
$M_2 \simeq M_{\chi_1^+}$

• $M_1 \simeq \dfrac{[M_{N_2}(M_{N_2}^2 - \mu^2 - m_Z^2) - s_W^2 m_Z^2 \mu\, m_Z \sin 2\beta]}{M_{N_2}^2 - \mu^2}$ for sufficiently large M_2

- Complex parameter case: roughly similar (except additional input)
↳ $\phi_L \simeq Z\chi_1^+\chi_2^+$

$$|\mu^2| = \frac{1}{2}\left[\Sigma - 2m_W^2(1+\cos 2\beta) - \Delta\cos(2\phi_L)\right]$$
$$M_2^2 = \frac{1}{2}\left[\Sigma - 2m_W^2(1-\cos 2\beta) + \Delta\cos(2\phi_L)\right]$$
$$\begin{matrix}\Sigma \\ \Delta\end{matrix} = m_{\chi_2^+}^2 \pm m_{\chi_1^+}^2$$

$$\cos\phi_\mu = \frac{\Delta - (M_2^2 - |\mu|^2)^2 - 4m_W^4\cos^2 2\beta - 4m_W^2(M_2^2+|\mu|^2)}{8\,m_W^2\sin 2\beta\; M_2|\mu|}$$

BUT $|\cos\phi_\mu| \leq 1 \rightarrow$ Constraints among $\{\phi_L, m_{\chi_1^+}, m_{\chi_2^+}\}$! :

$$\eta_{min} - 2m_W^2\cos 2\beta \lesssim \Delta\cos 2\phi_L \lesssim \eta_{max} - 2m_W^2\cos 2\beta$$
$$+\; -\eta_{max} - 2m_W^2\cos 2\beta \lesssim \Delta\cos 2\phi_L \lesssim -\eta_{min} - 2m_W^2\cos 2\beta$$

$$\eta_{\substack{min\\max}} = \sqrt{(\Sigma - 2m_W^2)^2 - 4(m_{\chi_1^+}m_{\chi_2^+} \pm m_W^2\sin 2\beta)^2}$$

- Neutralino sector:
same trick with the 4 invariants (but ! for $M_N M_N^\dagger$)

2 masses M_{N_i}, M_{N_j} (any) as input

→ quadratic system for $M_1 = \{\mathrm{Re}\,M_1, \mathrm{Im}\,M_1\}$

$$\mathrm{Re}\,M_1 = \frac{-b \pm\sqrt{b^2-4ac}}{2a} \quad \text{real ! → Constraints}$$

NB1: Constraints are actually related to the fact that not all the M_N entries are complex ! ($m_Z s_W \sin\beta, \ldots$)

NB2: ⇒ 2 sol. for $|M_1|, \varphi(M_1)$ in general

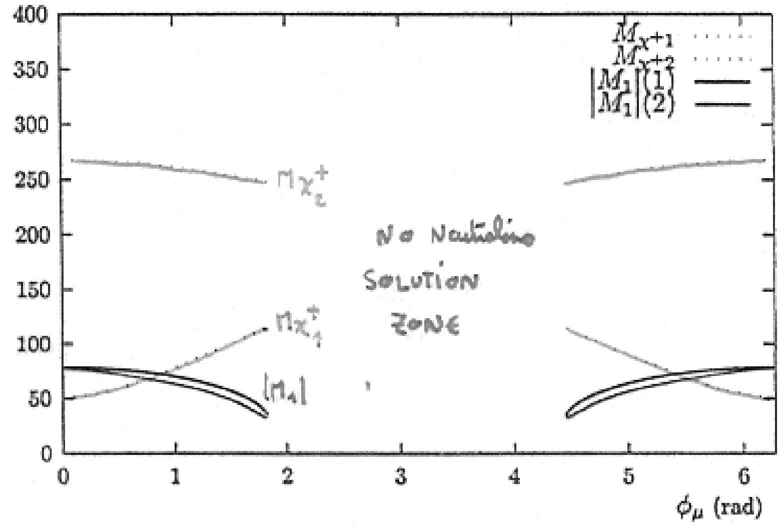
BUT are possibly distinguishable from $\sigma(\chi_1^0\chi_2^0)$ consistency → fig.

Note: we obtain $\sigma(e^+e^-\rightarrow\chi_i^0\chi_j^0) = f(m_{\chi_{1,2}^+}, \phi_L; M_{N_i}, M_{N_j}) + m_{\tilde{e}_L,\tilde{e}_R}$

→ severe consistency constraints !

[phases fully determined in Z_{ij} diag. matrix elts → e.g. $Z\chi_i^0\chi_j^0$]

1) Neutralino inversion only: assume $|\mu|, \phi_\mu, M_2$ input



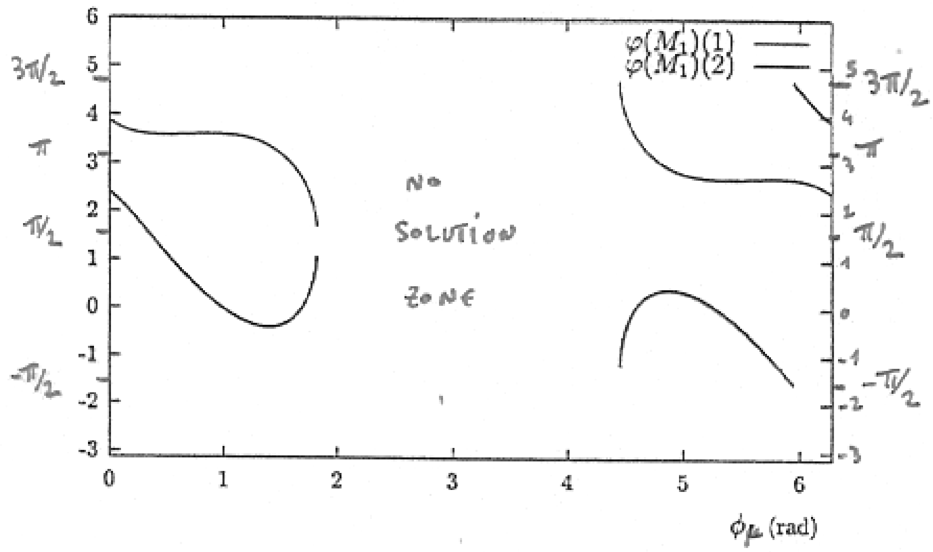
$|\mu| = 100$ GeV ; $M_2 = 120$ GeV

$0 < \phi_\mu < 2\pi$

$\tan\beta = 2$

$M_{N_1} = 40$ GeV

$M_{N_2} = 80$ GeV



$|\mu| = 100$ GeV

$M_2 = 120$ GeV

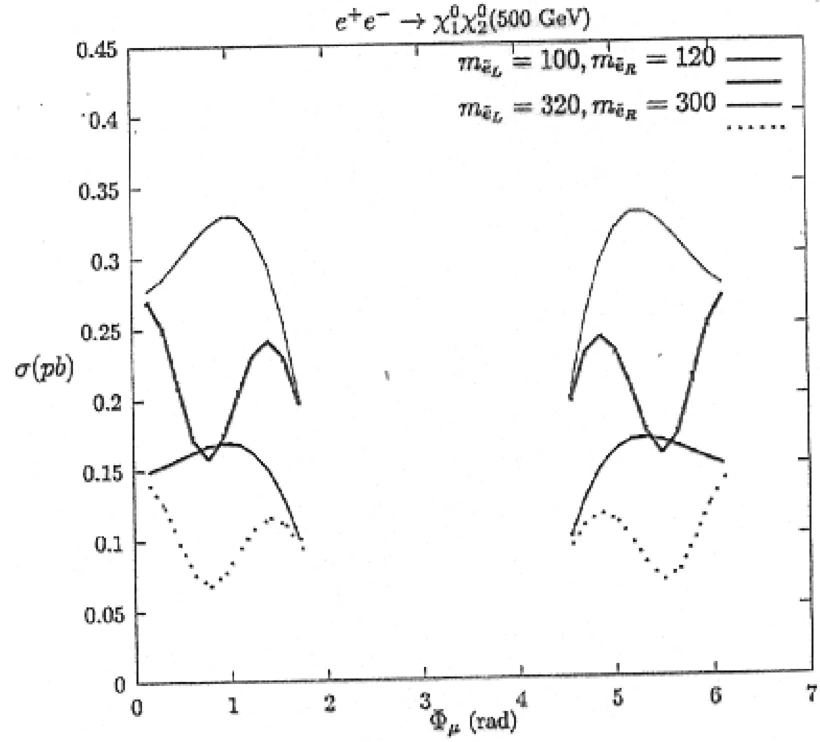
$\tan\beta = 2$

$M_{N_1} = 40$ GeV

$M_{N_2} = 80$ GeV

$\chi_1^\circ \chi_2^\circ$ production : complex case

(NB checked good agreement with real case calculations : Bartl, Fraas, Majerotto '8[illegible]
Ambrosanio, Mele, 9[illegible]



$M_2 = 120$ GeV

$|\mu| = 100$ GeV

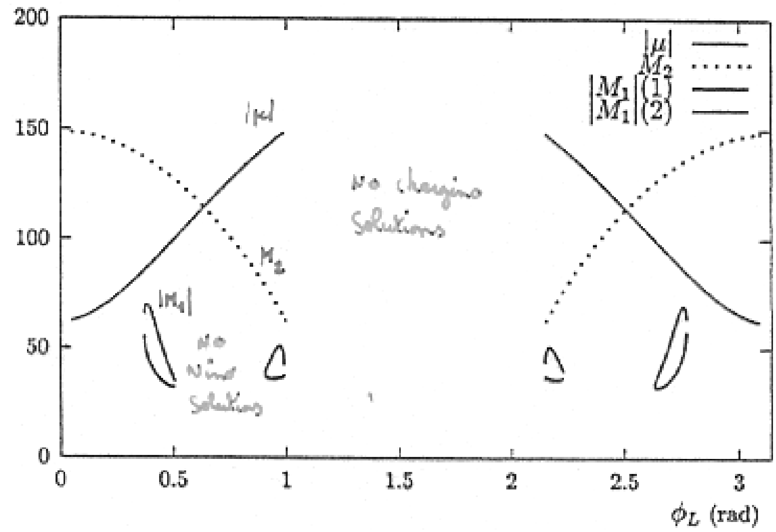
$\tan\beta = 2$ GeV

$M_{\chi_1^\circ} = M_{N_1} = 40$ GeV

$M_{\chi_2^\circ} = M_{N_2} = 80$ GeV

2) Complete inversion: $M_{\chi^\pm_{1,2}}, \phi_L$; $M_{N_{1,2}}$ input (+ $\tan\beta$)

$|\mu|, M_2, \varphi(\mu)$; $|M_1|, \varphi(M_1)$ output



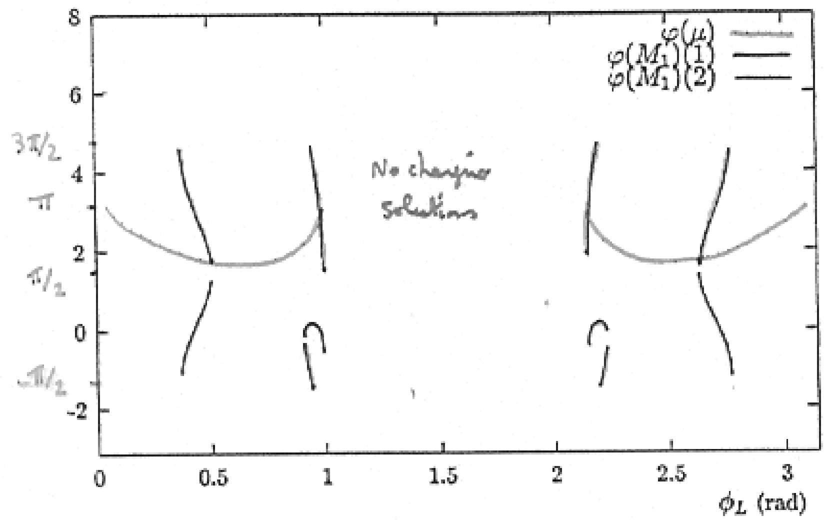
$M_{\chi_1^+} = 80$ GeV

$M_{\chi_2^+} = 180$ GeV

$\tan\beta = 2$

$M_{N_1} = 40$ GeV

$M_{N_2} = 80$ GeV



$M_{\chi_1^+} = 80$ GeV

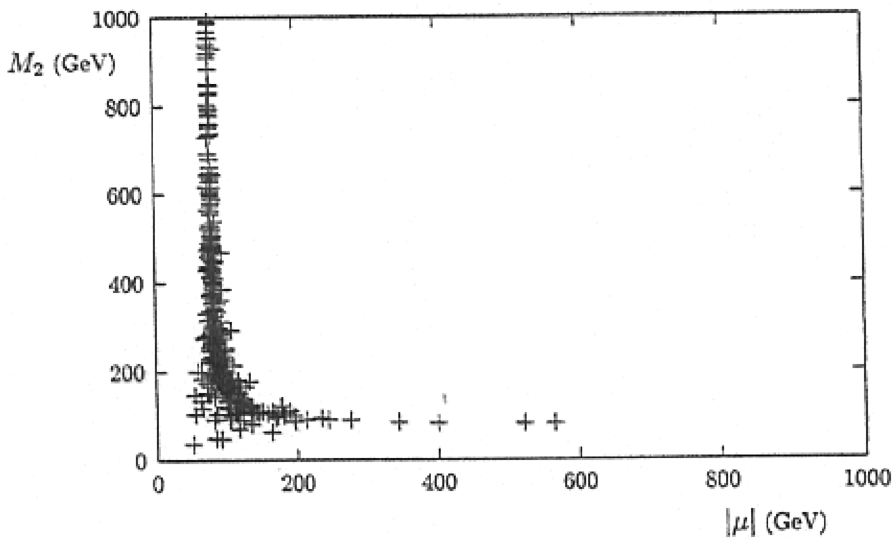
$M_{\chi_2^+} = 180$ GeV

$\tan\beta = 2$

$M_{N_1} = 40$ GeV

$M_{N_2} = 80$ GeV

(Preliminary)



$M_{\chi_1^\pm} = 80$ GeV

80 GeV $< M_{\chi_2^\pm} <$ 1 TeV

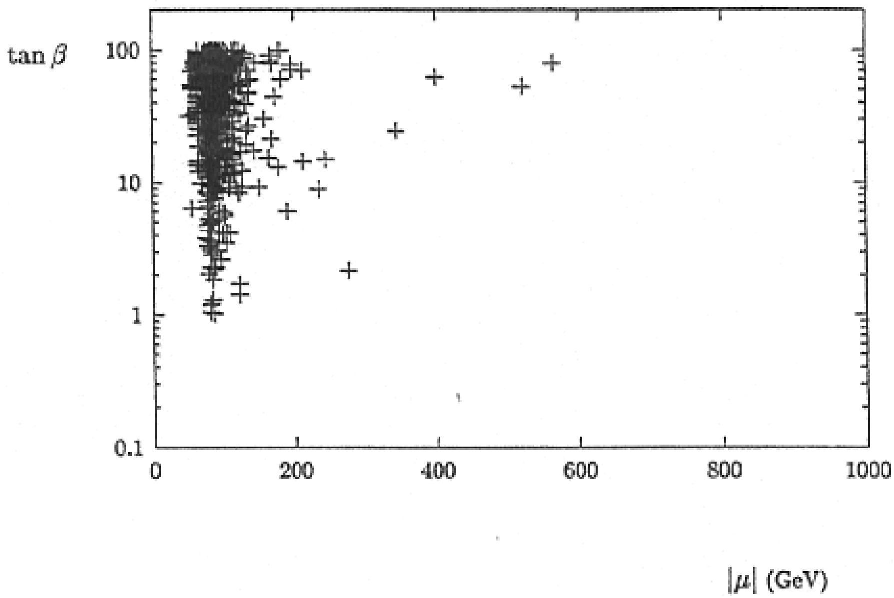
$0 < \phi_L < 2\pi$

$M_{N_1} = 40$ GeV

40 GeV $< M_{N_2} <$ 1 TeV

$1 < \tan\beta < 10^2$

(Preliminary)



$M_{\chi_1^+} = 80$ GeV

80 GeV $< M_{\chi_2^+} <$ 1 TeV

$0 < \phi_2 < 2\pi$

$M_{N_1} = 40$ GeV

40 GeV $< M_{N_2} <$ 1 TeV

$1 < \tan\beta < 100$

CONCLUSION

- SYSTEMATIC analytic INVERSION of ino spectrum obtained
- Relatively simple expressions → Fast algorithm
(even for full complex case: main routine $\lesssim$ 1 page Fortran code)

- Extend and complete more systematic analysis of $\sigma(\chi_1^+ \chi_2^+)$ Choi et al '98
(heavily relies on such analysis once real data available; BUT BEFORE: ANY SIMULATION should be easy with our algorithm)

- Perspective: more systematic simulations

  e.g.
  - exp errors (on masses) propagation?
  - implementation of present exp. bounds
  - implementation of more model-dependent assumption

(→ Numerical code (soon) available on request)

- SHOULD GIVE a BETTER INSIGHT in the non-trivial correlations among charginos/Neutralino parameters as implied by $\mathcal{L}_{MSSM}$