Determining SUSY Parameters in Chargino Pair-Production in e^+e^- Collisions

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I report "mainly" on

$\boxed{\text{CDKZ}}$: SYC, Djouadi, Dreiner, Kalinowski, Zerwas, EPJC '99
SYC, Djouadi, Song, Zerwas, hep-ph/9812236

$\boxed{\text{KM}}$: Kneur, Moultaka, PRD '99

and also mention a few related recent works :

- Moortgat-Pick, Fraas, Bartl, Majerotto, EPJC '99
- Moortgat-Pick, Fraas, hep-ph/9904209
- Lafage et al., hep-ph/9810504

in demonstrating how one can determine SUSY parameters in a model independent way in the chargino [+ neutralino] systems in

$\boxed{R_p\text{-preserving MSSM}}$

May 3rd, 1999 @ Sitges / Spain

$\boxed{\text{Chargino structure}}$

- **Chargínos $\tilde{\chi}^{\pm}_{1,2}$** : mix $\tilde{W}^{\pm} \oplus \tilde{H}^{\pm}_{1,2}$

<1> $$\begin{bmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{bmatrix} \begin{bmatrix} \cos\phi_R & e^{-i\beta_R}\sin\phi_R \\ -e^{i\beta_R}\sin\phi_R & \cos\phi_R \end{bmatrix} \begin{bmatrix} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & |\mu|e^{i\Phi_\mu} \end{bmatrix} \begin{bmatrix} \cos\phi_L & e^{-i\beta_L}\sin\phi_L \\ -e^{i\beta_L}\sin\phi_L & \cos\phi_L \end{bmatrix}^{\dagger} = \mathrm{diag}\left(m_{\tilde{\chi}^{\pm}_1}, m_{\tilde{\chi}^{\pm}_2}\right)$$

$$m^2_{\tilde{\chi}^{\pm}_{1,2}} = \frac{1}{2}\left[M_2^2 + |\mu|^2 + 2m_Z^2 \mp \Delta\right] \qquad \cos 2\phi_{L,R} = -\frac{M_2^2 - |\mu|^2 \mp 2m_W^2\cos 2\beta}{\Delta}$$

$$\Delta = \left[(M_2^2-|\mu|^2)^2 + 4m_W^4\cos^2 2\beta + 4m_W^2(M_2^2+|\mu|^2) + 8m_W^2 M_2|\mu|\sin 2\beta\cos\Phi_\mu\right]^{1/2}$$

$\boxed{\text{conceptual connection}}$

- **fundam. param.** — **physical param.** — **physical observables**

fundam. param.:

M_2

$|\mu|$

Φ_μ

$\tan\beta$

$\oplus$ $|M_1|$, Φ_1 } neutralino

physical param.: $m_{\tilde{\chi}^{\pm}_{1,2}}$, $\cos 2\phi_{L,R}$, ⋮, $\oplus\, m_{\tilde{\chi}^0_i}$

KM (real)

CDKZ (complex)

physical observables: masses, σ_{ij}, spin correlations, A^{ij}_{LR}

CDKZ (11) (real)

" (complex) all modes

SUSYGEN, GRACE SUSY

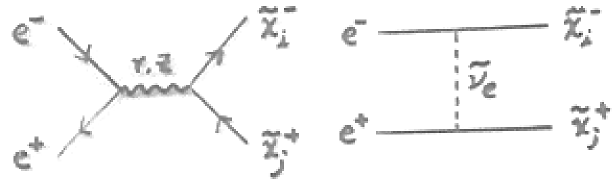
ISASUSY, SPythia, Pandora ⋯

Tsukamoto et al.

Feng et al.

independent of detailed decay pattern

CDKZ



$$= Q_{\alpha\beta}\, J^{ep}_{\mu}(\alpha)\, J^{\tilde{\chi}\mu}(\beta)$$

① Low energy : $\tilde{\chi}_1^- \tilde{\chi}_1^+$ + "real couplings"

$$Q_{LL} = 1 + \frac{Z}{s_W^2 c_W^2}\left(s_W^2 - \frac{1}{2}\right)\left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_L\right)$$

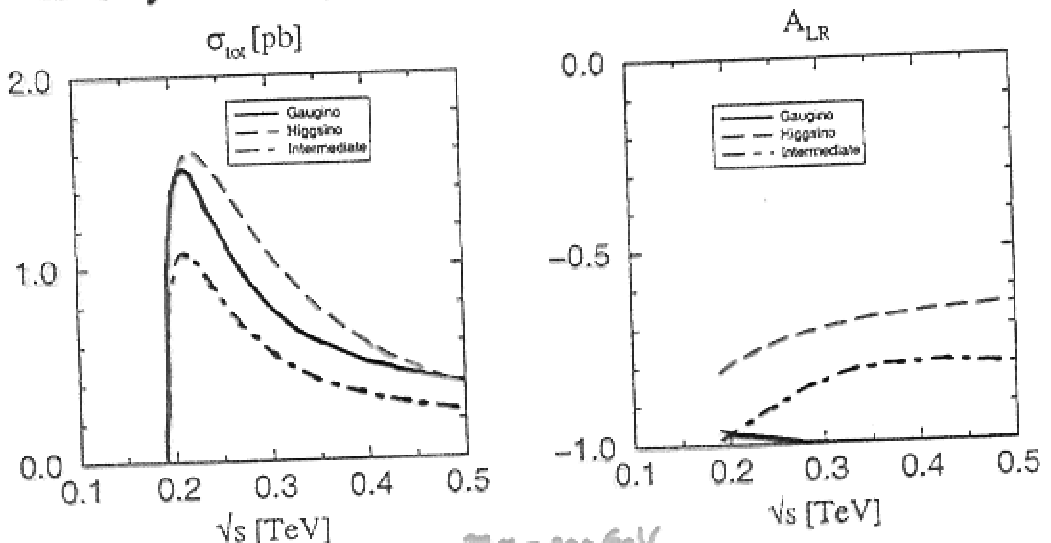
$\langle 2 \rangle$

$$Q_{LR} = 1 + \frac{Z}{s_W^2 c_W^2}\left(s_W^2 - \frac{1}{2}\right)\left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_R\right) + \frac{\tilde{D}}{4 s_W^2}\left(1 + \cos 2\phi_R\right)$$

⋮

gaugino	:	$M_2 = 81$	$\mu = -215$	GeV
higgsino	:	$M_2 = 215$	$\mu = -81$	GeV
mixed	:	$M_2 = 92$	$\mu = -93$	GeV

(a) mass from sharp spin-½ excitation curve at threshold



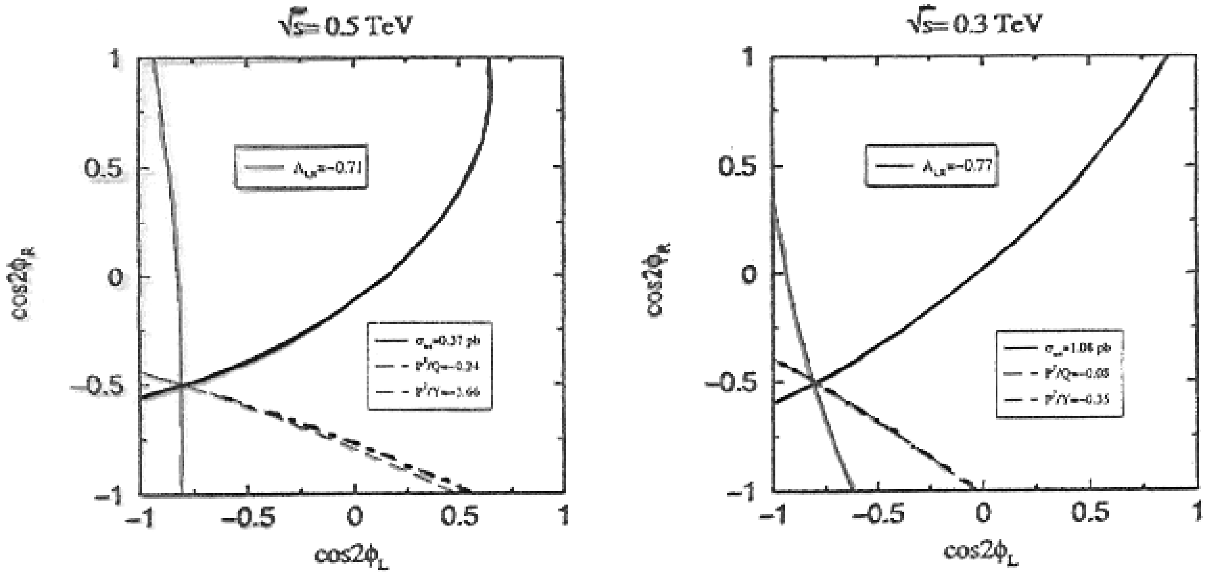
$m_{\tilde{\nu}} = 200$ GeV

(b) total cross section : quadratic in [$\cos 2\phi_L$, $\cos 2\phi_R$]

(c) $\left.\begin{array}{l}\tilde{\chi}_1^- \to W^- + \tilde{\chi}_1^0 \\ \tilde{\chi}_1^+ \to W^+ + \tilde{\chi}_1^0\end{array}\right] \xrightarrow[\Delta\varphi_*]{\cos\theta_*^\pm} \left[\begin{array}{ll}\text{Polarization} & : \langle P(\tilde{\chi}_1^\pm)\rangle = \mathcal{P} \\ \text{long. Pol. Corr.} & : \langle P_L(\tilde{\chi}_1^-) P_L(\tilde{\chi}_1^+)\rangle = Q \\ \text{trans. Pol. Corr.} & : \langle P_T(\tilde{\chi}_1^-) P_T(\tilde{\chi}_1^+)\rangle = Y\end{array}\right.$

(Moortgat-Pick et al.), Ghodbane

Come with analysis powers $\kappa_\pm$ → eliminate them by taking ratios $\mathcal{P}^2/Q$, $\mathcal{P}^2/Y$



⇨ unique solution : $m_{\tilde{\chi}_1^\pm}$, $\cos 2\phi_L$, $\cos 2\phi_R$, $m_{\tilde{\nu}} = 250$ GeV

<u>Fundamental parameters</u>

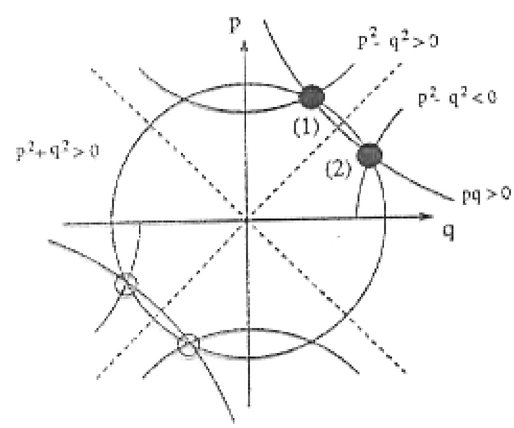
$$\tan\beta = \frac{p^2 - q^2 \pm 2\sqrt{\chi^2(p^2+q^2+2-\chi^2)}}{(\sqrt{1+p^2} - \sqrt{1+q^2})^2 - 2\chi^2}$$

<3> $$M_2/\mu = \frac{m_W}{\sqrt{2}}\left[(p\pm q)s_\beta - (p\mp q)c_\beta\right]$$

$$\cos 2\phi_R \gtrless \cos 2\phi_L \leftrightarrow \tan\beta \gtrless 1$$

$$p/q = \cot(\phi_R \mp \phi_L)$$

$$\chi^2 = m^2_{\tilde{\chi}_1^\pm}/m_W^2$$



⇨ At most 2-fold ambiguity

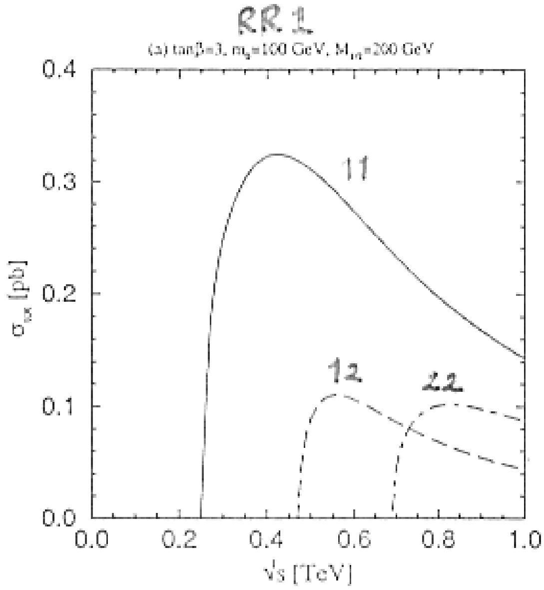
② High energy : $\tilde{\chi}_i^- \tilde{\chi}_j^+ \; [i,j=1,2]$ + "complex couplings"

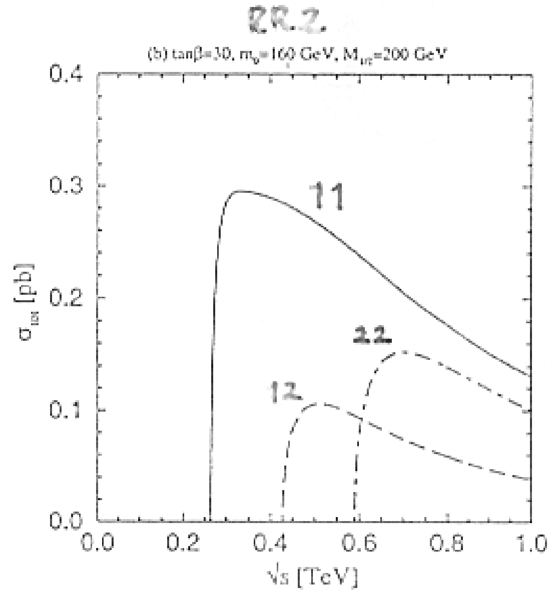
RR1 :	$\tan\beta = 3$,	$M_2 = 152$,	$\mu = 316$,	$m_{\tilde{\nu}} = 166$ GeV
RR2 :	$\tan\beta = 30$,	$M_2 = 150$,	$\mu = 263$,	$m_{\tilde{\nu}} = 206$ GeV

Ambrosanio et al.

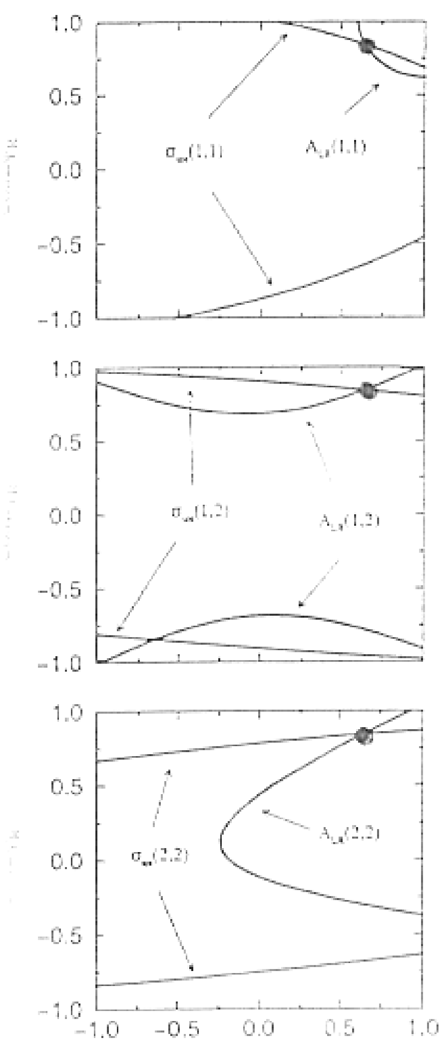
(a) masses

$m_{\tilde{\chi}_1^\pm}, \; m_{\tilde{\chi}_2^\pm}$





RR1 ($\sqrt{s} = 800$ GeV)



(b) Com. of σ_{ij} in 2 channels
Com. of σ_{ij}, A_{LR}^{ij} in 1 channel } "Unique solution"

⇨ If NOT, more complex combinations

⇨ Unique physical para. $m_{\tilde{\chi}_1^\pm}$ $m_{\tilde{\chi}_2^\pm}$ $\cos 2\phi_L$ $\cos 2\phi_R$

"3 SUM RULES"

Fundamental parameters $\Delta = m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm}$

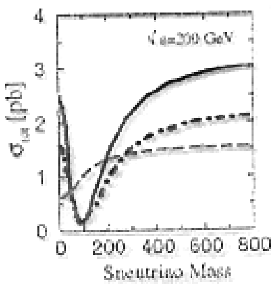
$$\tan\beta = \sqrt{\frac{4m_W^2 + \Delta(\cos 2\phi_R - \cos 2\phi_L)}{4m_W^2 - \Delta(\cos 2\phi_R - \cos 2\phi_L)}}$$

$$M_2/|\mu| = \frac{1}{2}\sqrt{2(m^2_{\tilde{\chi}_2^\pm} + m^2_{\tilde{\chi}_1^\pm} - 2m_W^2) \mp \Delta(\cos 2\phi_R - \cos 2\phi_L)}$$

$$\cos\Phi_\mu = \frac{\Delta^2 - (M_2^2 - |\mu|^2)^2 - 4m_W^2(M_2^2 + |\mu|^2) - 4m_W^4\cos^2 2\beta}{8m_W^2 M_2 |\mu| \sin 2\beta}$$

⇨ Unique determination of fundam. SUSY parameters ???

Remarks



$m_{\tilde\nu}$

① $m_{\tilde\nu_e}$: $e^+e^-_L \longrightarrow \tilde e_L \tilde e_L^*$ $\boxed{m^2_{\tilde\nu_e} - m^2_{\tilde e} = m_W^2 \cos 2\beta}$

$e^+e^-_p \to \tilde\chi_1^+\tilde\chi_1^- \to \tilde\chi_1^0 e^- \bar{\tilde\nu}_e$ $\tilde\nu \to e^- \tilde\chi^+$

(Moortgat-Pick & Fraas) Peskin

② $\mathrm{sgn}(\sin\Phi_\mu)$: $O_T = \vec p_{e^-}\cdot(\vec p_{\ell^+}\times\vec p_{\ell^-})$ $\ell^\pm$ from $\tilde\chi_1^\pm$

$\boxed{\mathrm{KM}}$ + "real couplings"

Goal: $\tan\beta + \begin{cases} S1: m_{\tilde\chi_1^\pm},\ m_{\tilde\chi_2^\pm},\ \pm m_{\tilde\chi_1^0} \\ S2: m_{\tilde\chi_1^\pm},\ \pm m_{\tilde\chi_1^0},\ \pm m_{\tilde\chi_2^0} \end{cases} \Rightarrow M_1, M_2, \mu$

based on M_c and the neutralino mass matrix M_N

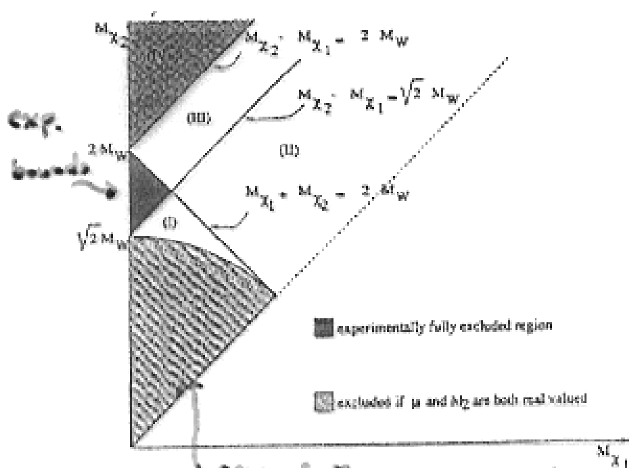
4 invariants

<5> $M_N = \begin{array}{c} \tilde B \\ \tilde W^0 \\ \tilde H_1^0 \\ \tilde H_2^0 \end{array}\begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ & & 0 & -\mu \\ (\mathrm{sym.}) & & & 0 \end{pmatrix} \Rightarrow$ $\mathrm{tr}\,M_N,\ \frac{1}{2}[(\mathrm{tr}\,M_N)^2 - \mathrm{tr}\,M_N^2]$, $\frac{1}{6}[(\mathrm{tr}\,M_N)^3 - 3\,\mathrm{tr}\,M_N\,\mathrm{tr}\,M_N^2 + 2\,\mathrm{tr}\,M_N^3]$, $\det M_N$

"linear in M_1 and M_2"

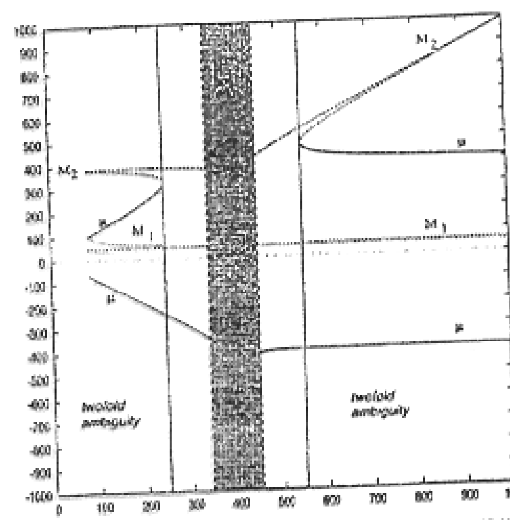
S1

$\Rightarrow M_1 = M_1(\tan\beta;\ M_2, \mu : \pm m_{\tilde\chi_1^0})$ completely solvable !



one of M_2, μ : NOT real

up to a 2-fold ambiguity



$M_{\tilde\chi_2^\pm}$

"two-fold" ambiguity

Remark:

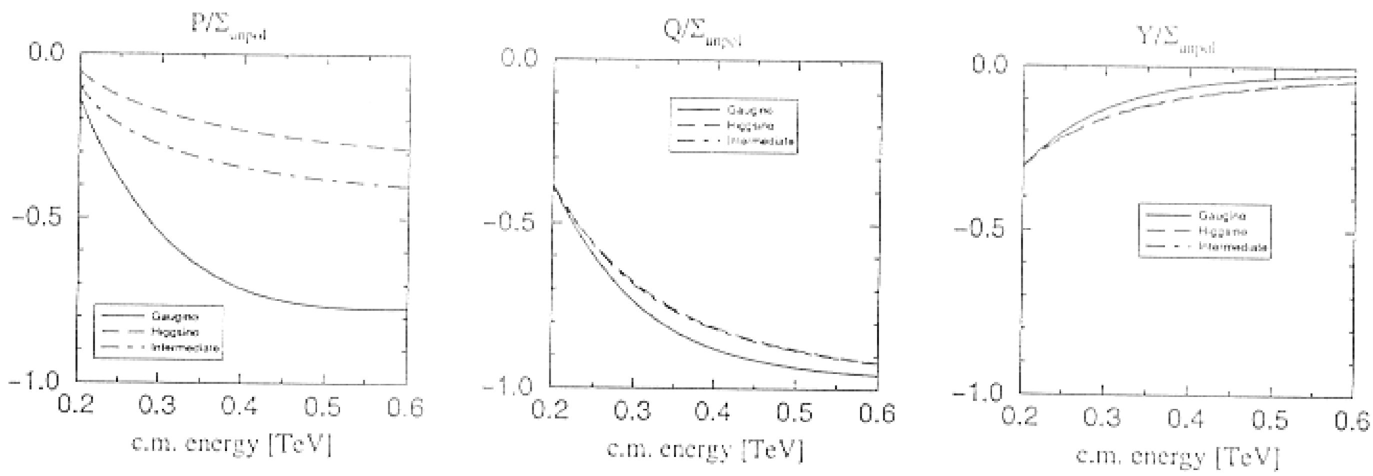
① Determination of the two-fold ambiguity pattern and the general trend of the solutions

② No phases yet → Kneur

$\boxed{\text{Expected Errors}}$

(a) Sensitivity to SUSY Parameters

Normalized Spin–Spin Correlations



$$\Rightarrow \quad \left.\begin{array}{l} \sigma_{tot},\ P,\ A_{LR} : \text{sensitive} \\ Q,\ Y : \text{insensitive} \end{array}\right\} \rightarrow P^2/Q,\ P^2/Y : \text{sensitive}$$

consistent with Lafage et al. '9

(b) $\boxed{1\sigma \text{ errors on } \sigma_{ij},\ A_{LR}^{ij} \text{ and } \Delta m_{\tilde{\chi}^\pm_i} = 0.1\ \text{GeV}}$

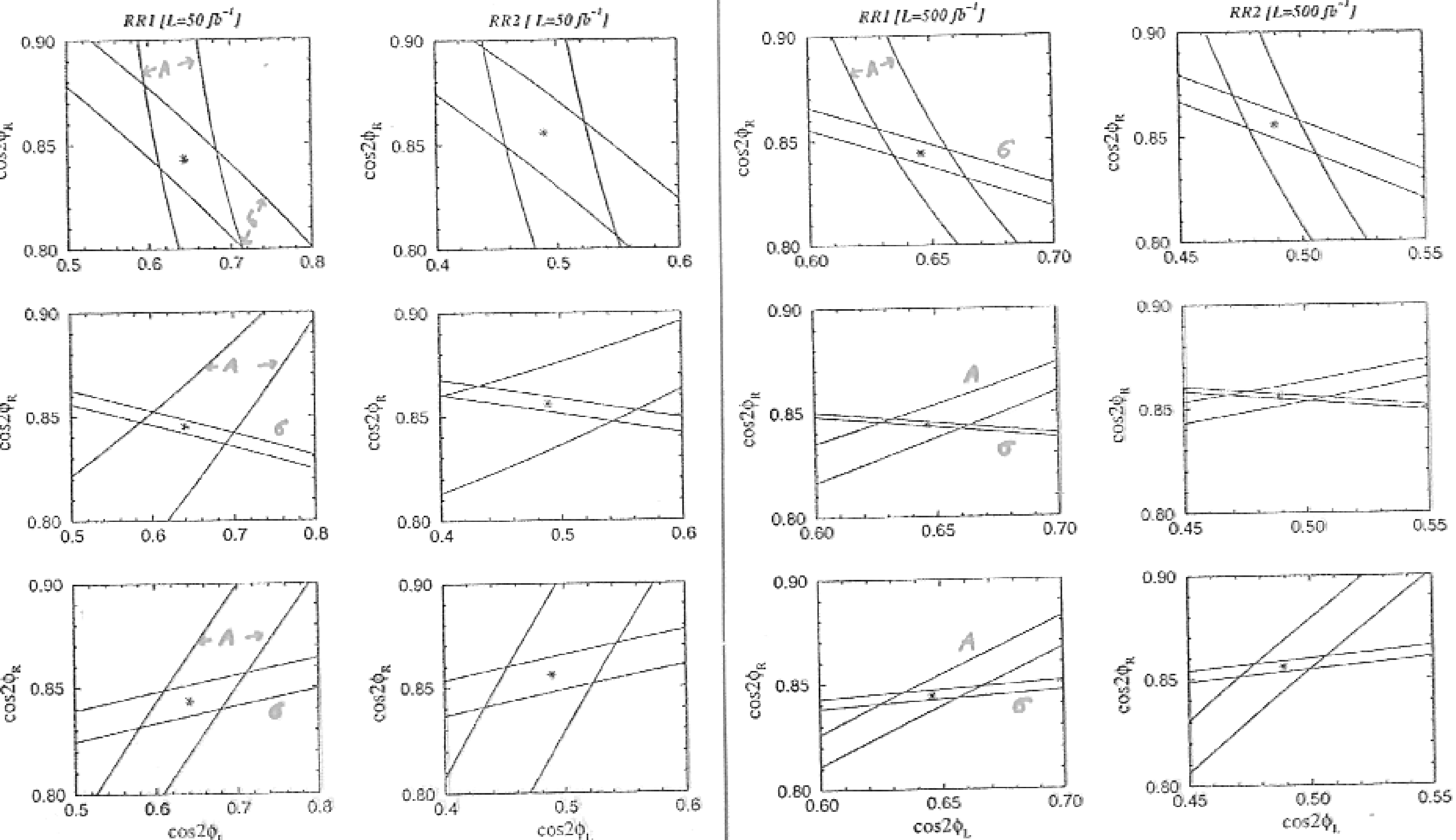
$\mathcal{L} = 500\,fb^{-1}$ $\Rightarrow$ $\Delta(\cos 2\phi_L) = 0.02$, $\Delta(\cos 2\phi_R) = 0.005$

	RR1	RR2
M_2	152 ± 1.8	150 ± 1.2
μ	316 ± 0.9	263 ± 0.7
$\tan\beta$	3 ± 0.7	30 ± 380 !

→ Neutralinos do not help

- $\tilde{\chi}^\pm_i, \tilde{\chi}^0_i$: sym. under $\beta \leftrightarrow \pi/2 - \beta$ $\Rightarrow$ $\cos 2\beta$, $\sin 2\beta$
- large $\tan\beta$ → $\sin 2\beta \simeq 0$, $\cos 2\beta \simeq -1$

1·6



RR1 [L=50 fb^{-1}]
RR2 [L=50 fb^{-1}]
RR1 [L=500 fb^{-1}]
RR2 [L=500 fb^{-1}]
$\cos 2\phi_R$
$\cos 2\phi_L$
A
σ

- Different strategy : $\boxed{M_2,\ \mu} \longrightarrow \boxed{\cos 2\beta} \longrightarrow \boxed{\mathrm{sgn}(\mu) + \sin 2\beta} \Rightarrow$ calculate $\tan\beta$

- M_1 : Solve $x^4 - \mathrm{tr} M_N x^3 + \frac{1}{2}[(\mathrm{tr} M_N)^2 - \mathrm{tr} M_N^2] x^2 - \frac{1}{6}[(\mathrm{tr} M_N)^3 - 3\,\mathrm{tr} M_N\, \mathrm{tr} M_N^2 + 2\,\mathrm{tr} M_N^3] x - \det M_N = 0$
  for $x = \pm m_{\tilde\chi_1^0}$

$\boxed{\text{Errors}}$ $\sqrt{s} = 800$ GeV

RR1 ($\tan\beta=3$)	$\mathcal{L}=50\,fb^{-1}$	$\mathcal{L}=500\,fb^{-1}$	RR2 ($\tan\beta=30$)	$\mathcal{L}=50\,fb^{-1}$	$\mathcal{L}=500\,fb^{-1}$
$m_{\tilde\chi^\pm} = 128/345$ GeV	0.1 GeV	0.1 GeV	132/285 GeV	0.1 GeV	0.1 GeV
$\cos 2\phi_L = 0.65$	0.12	0.02	0.49	0.12	0.02
$\cos 2\phi_R = 0.84$	0.02	0.005	0.86	0.02	0.005
$M_2 = 152$ GeV	10.3	1.75	150 GeV	7.03	7.20
$\mu = 316$ GeV	4.96	0.85	263 GeV	4.0	0.68
$\cos 2\beta = -0.8$	0.49	0.08	−0.9978	0.33	0.056
$\sin 2\beta = 0.6$	0.34	0.058	0.0666	0.19	0.033
$\tan\beta = 3$	1.4–7.6	2.7–3.4	30	7.6–∞	20.2–59.6
$M_1 = 76/-67$ GeV	3.5/2.4	0.64/0.45	75.4/−74	1.96/1.46	0.40/032

* large $\mathcal{L}$: crucial for $\tan\beta$?? $\xrightarrow[\text{otherwise}]{}$ Go to Higgs-related sectors

Sopczak, Nojiri
Feng & Murai
⋮

Conclusions

- $\underline{\text{CDKZ}}$: σ, A_{LR} Spin-corr. $\Rightarrow$ $m_{\tilde{\chi}^{\pm}_{1,2}}$ $\cos 2\phi_{L,R}$ $\Rightarrow$ Complete determination
- $\underline{\text{KM}}$: masses $\Rightarrow$ "analytic" determination
- $\underline{\text{Expected Errors}}$: Controllable except for large $\tan\beta$

  ☼ Q. a better way to determine $\tan\beta$?

  - $\tan\beta$?
  - Full determ. of SUSY parameters ?
    1. w/o radiative corrections
    2. w "
       → Diaz