

Physics and Experiments with
Future Linear e^+e^- Colliders

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Top Quark Mass
Definitions and the
 $t\bar{t}$ threshold
cross section

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Content

- Outline of the problem:
 $\sigma(e^+e^- \rightarrow t\bar{t})$ @ threshold
and the Pole Mass scheme
- The source of the problem:
IR sensitivity and
strong correlations
- The way out: Short
Distance Mass definitions,
▶ Impact on the threshold
cross section
- Conclusions

I. $\sigma(e^+e^- \rightarrow t\bar{t})$ @ threshold

→ ... talks of M. Peskin, Y. Sumino

- threshold scan best suited for a precise determination of the top quark mass m_t .
▷ very small exp. errors!

● But:

How good is our theoretical prediction? Are there large
▷ systematic uncertainties?

Recent NNLO corrections \Rightarrow

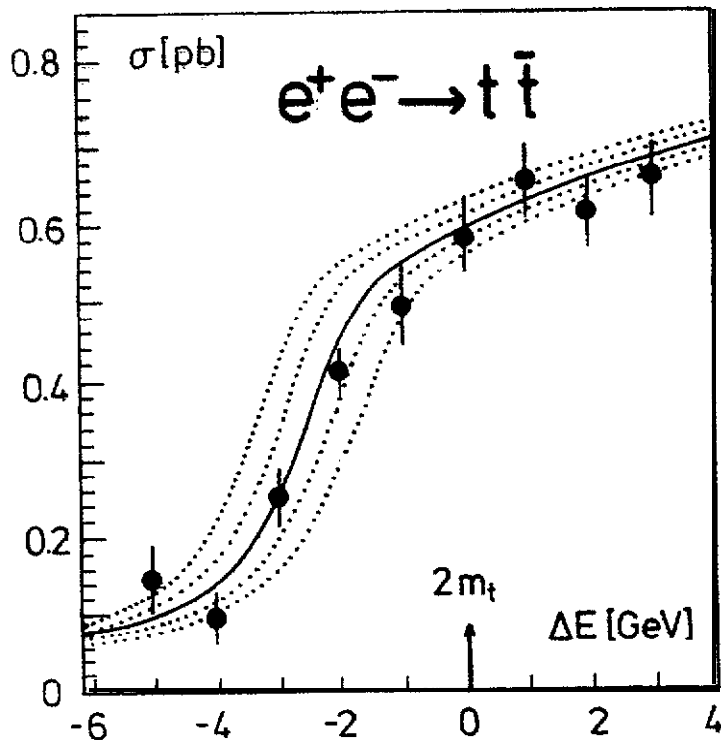
large

- LO \rightarrow NLO \rightarrow NNLO:
Peakshift
- μ -dependence

Exp. analysis:

Martinez
Miguel
Schafer

e⁺e⁻



9 point scan;
5 or even 3
points might
be the better
strategy.
Analyses under
way...

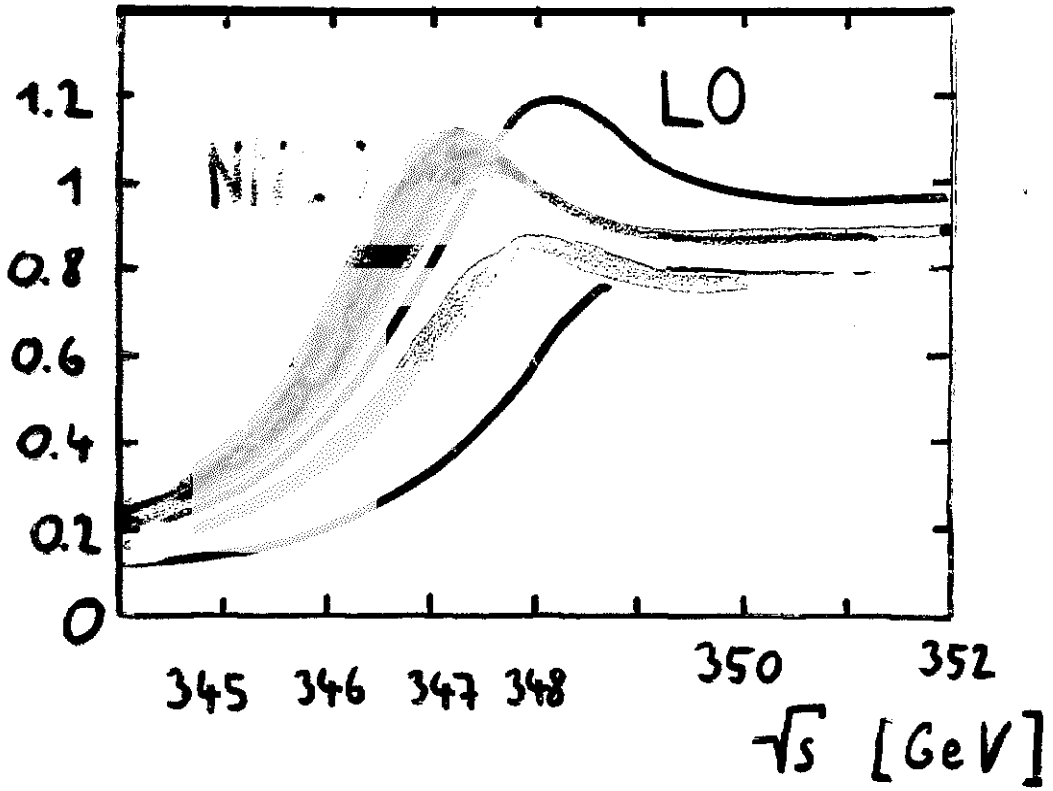
$$\sqrt{s} = 2m_t$$

Figure 7: Excitation curve of the top quarks including initial-state radiation and beamstrahlung. The errors of the data points correspond to an integrated luminosity of $\int \mathcal{L} = 50 \text{ fb}^{-1}$ in toto. The dotted curves indicate shifts of the top mass by 200 and 400 MeV. Ref.[29, 40].

latest TESLA Design : $\sim 21000 \text{ } t\bar{t} / 50 \text{ fb}^{-1}$

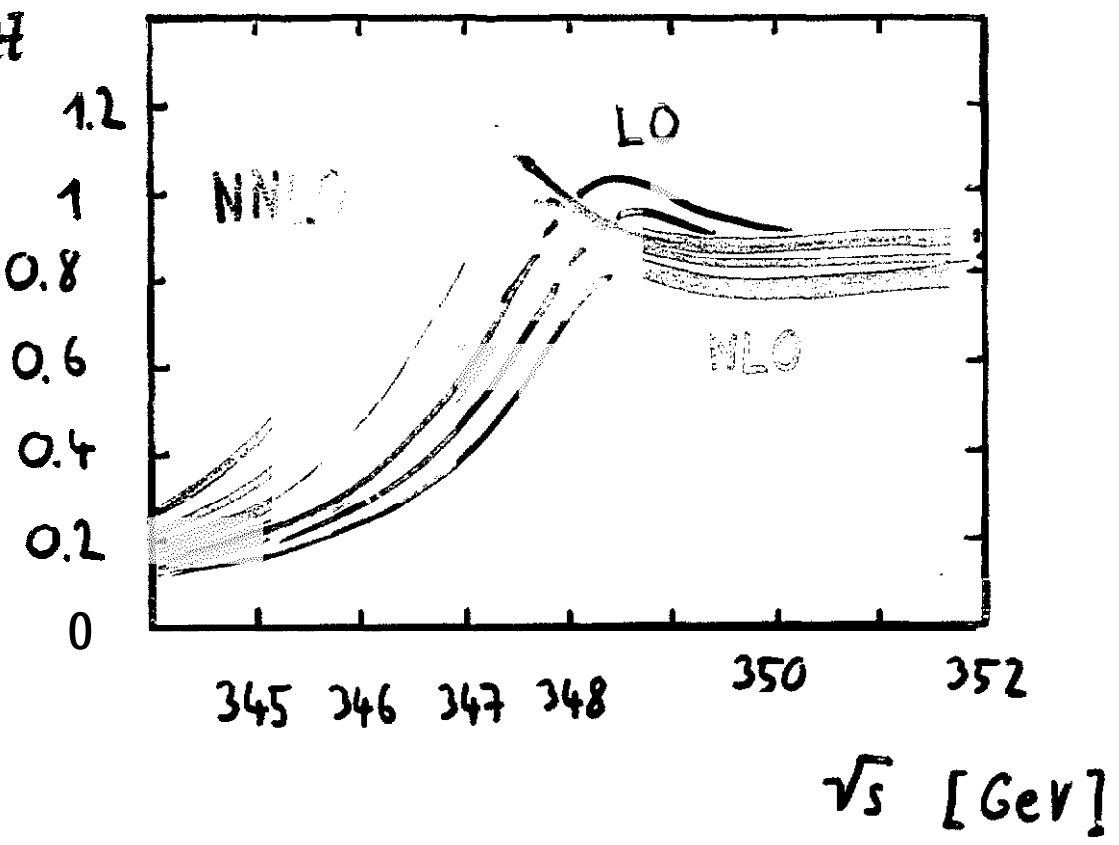
$\mu = 15 \dots 60 \text{ GeV}$, $\alpha_s(M_Z) = 0.118$

$R_{t\bar{t}}^v$



Pole Mass Scheme

R_{tt}^v



② The problem: IR sensitivity

and strong $\alpha_s \leftrightarrow m_t$ correlations
which mass?

● On the one hand:

* threshold regime, non-relativistic system,
large mass, formulation of effective
theories $\rightarrow m^{\text{pole}}$ is very useful

* m^{pole} is IR finite and gauge-independent
to all orders in pQCD. A. Kronfeld

● But on the other hand:

* m^{pole} is NOT an observable!

* defined only up to IR ambiguity
of $\mathcal{O}(\Lambda_{\text{QCD}})$ (\rightarrow confinement).

* the large width $\Gamma_t \sim 1.5 \text{ GeV}$ does
NOT protect m^{pole}

- Energy of the 1S resonance is (in principle) an observable and unambiguous.

$$\rightarrow E_{\text{static}}(\tau) = 2m + V(\tau)$$

has no IR ambiguity :
cancellation of the leading IR Renormalization
contributions :

$$2 \cdot \text{[Diagram 1]} + \text{[Diagram 2]} = 0$$

(large order phenomenon)

A. Jentsch
A. Jentsch et al.

\Rightarrow use a scheme, where also the single contributions m , V , are NOT IR-sensitive ("short distance")

$$E_{\text{static}}(\tau) = 2m_{\tau}^{\text{pole}} + V(\tau)$$

$$= 2m_{\tau}^{\text{SD}} + V(\tau)$$

- But: Cancellation of leading IR contributions is not sufficient to cure the strong $\alpha_s \leftrightarrow m_t$ correlations at "low" (NLO, NNLO) orders:

$$M_{\text{peak}} = 2M_t - \delta M_{\text{peak}}^{\text{LO}} - \delta M_{\text{peak}}^{\text{NLO}} - \delta M_{\text{peak}}^{\text{NNLO}, \{\beta_0, \text{rest}\}}$$

$\mu [\text{GeV}]$	$\delta M_{\text{peak}}^{\text{total}}$	δ^{LO}	δ^{NLO}	$\delta^{\text{NNLO}, \beta_0}$	$\delta^{\text{NNLO}, \text{resi}}$
15	2.8	1.89	0.31	0.24	0.37
30	2.58	1.26	0.75	0.31	0.27
60	2.39	0.69	1.09	0.40	0.20

$$(\alpha_s = 0.118, m_t^{\text{pole}} = 175 \text{ GeV})$$

→ leading IR contributions are only half of the game.

Subtraction of non-IR contributions needed to "stabilize the peak".

III. Short Distance Mass Definitions

- "Potential Subtracted" Mass $m^{PS}(\mu_f)$:

subtract universal IR-contributions ...
from the potential:

$$m^{PS}(\mu_f) = m^{pole} - \delta m(\mu_f)$$

$$\delta m(\mu_f) = -\frac{1}{2} \int_{|\vec{q}| < \mu_f} \frac{d^3 q}{(2\pi)^3} \tilde{V}(q)$$

$$V(\tau, \mu_f) = V(\tau) + 2 \delta m(\mu_f)$$

$$\Lambda_{QCD} \ll \mu_f \lesssim m_t C_F \alpha_s(\mu) \approx 30 \text{ GeV}$$

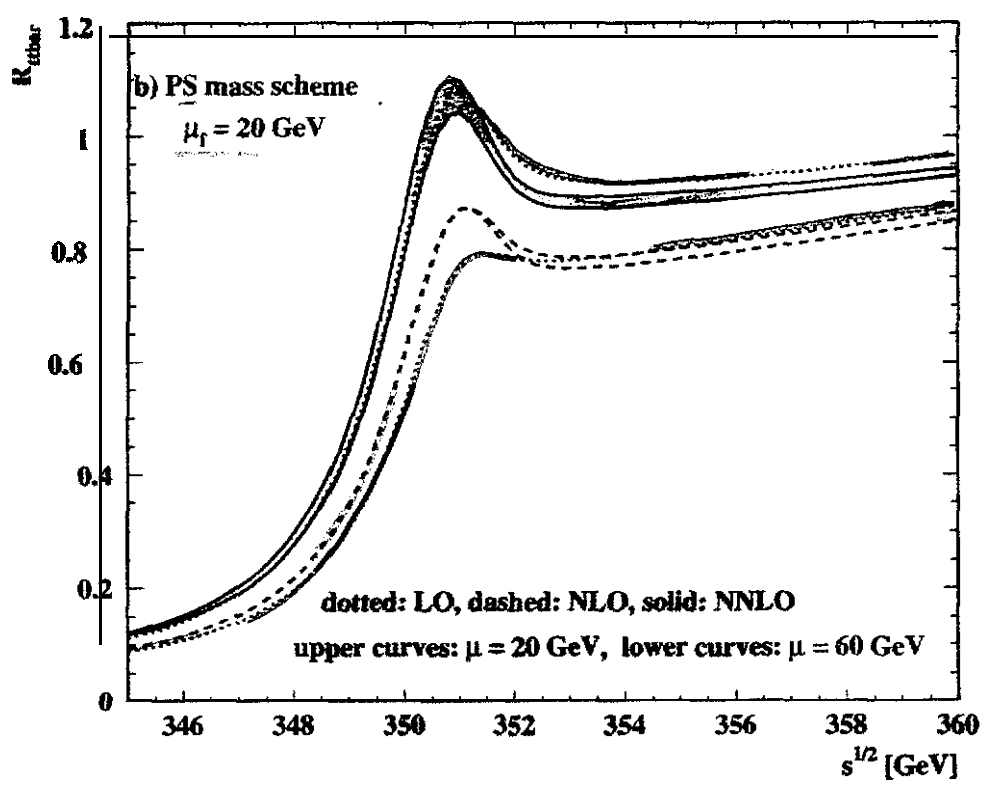
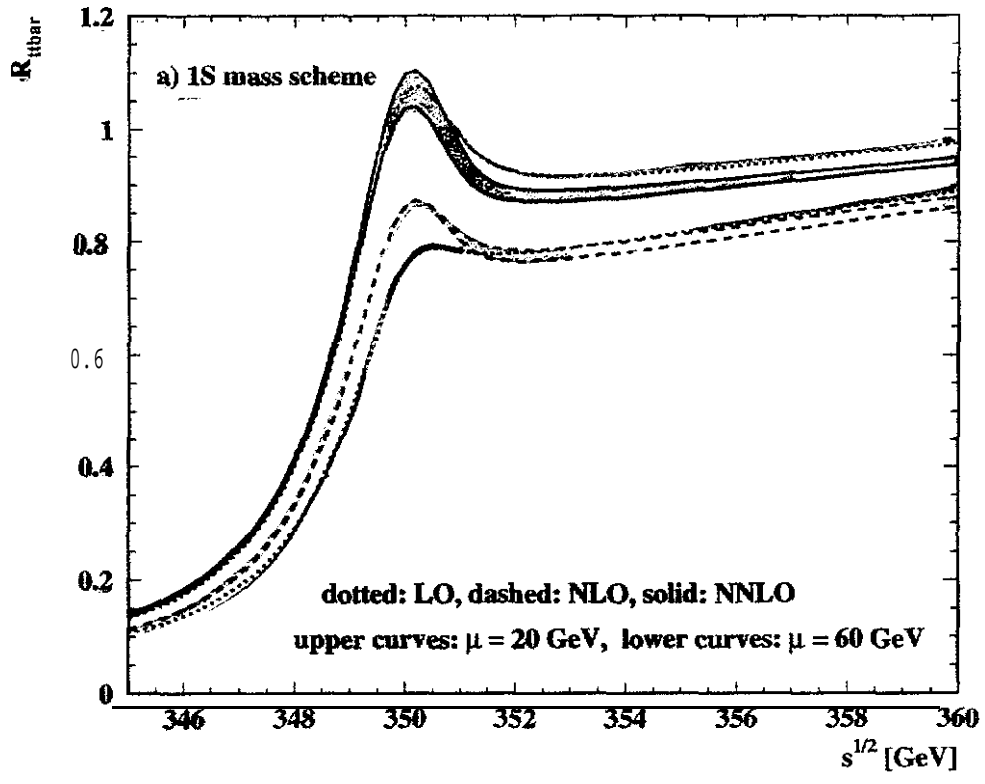
$\mu_f = 3 \text{ GeV}$: Results presented in ... Seminar's talk.

... ..

... ..

Use of m^{PS} in numerics à la Hoang + Teubner: ...

→ good stabilization of the peak.



LO
NLO
NNLO

→ m^{PS} can be extracted reliably
from $\sigma_{tot}(e^+e^- \rightarrow t\bar{t})$ @ threshold.

→ Relation to $m^{\overline{MS}}$ preferred at
high energies and in EW precision
observables:

assume $m_t^{\overline{MS}} (m_t^{\overline{MS}}) = 165.0 \text{ GeV}$:

$$m_t^{\text{pole}} = [165.0 + 7.6 + 1.6 + 0.6 \text{ (est.)}] \text{ GeV}$$

$$m_t^{PS}(\mu_f = 20 \text{ GeV}) = [165.0 + 6.7 + 1.2 + 0.3 \text{ (est.)}] \text{ GeV}$$

("est." = large $-\beta_0$ -limit)

● Similar to m^{PS} :

"low scale running mass" $m^{LS}(\mu_{LS})$:

→ Subtract the IR contributions from the heavy
quark self energy.

→ Universality of the leading IR sensitivity → similarity of m^{PS} , m^L

$$m^{LS}(\mu_{LS}) - m^{pole} = -\frac{16}{9} \frac{\alpha_s}{\pi} \mu_{LS} \left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{\mu_L}{m^2}\right) \right]$$

$$m^{PS}(\mu_{PS}) - m^{pole} = -\frac{4}{3} \frac{\alpha_s}{\pi} \mu_{PS} \left[1 + \mathcal{O}(\alpha_s) \right]$$

→ μ_{LS} (or μ_{PS}) can be chosen to achieve subtraction of non-IR corrections to the peak position at NLO, NNLO.

- $M^{1S} :=$ half of the perturbative mass of the fictitious ($\Gamma_t \rightarrow 0$) toponium 1S ground state

Heurich, 2011
M. J. ...

- close to the "quasi-observable"
1S - peak, but defined within PT.
 - subtraction of the same universal IR contribution as m^{PS}, m^{LS} .
 - no dependence on additional scale.
 - stabilization of the peak in σ_{tot} "by construction":
- Heurich, 2011
- normalization - uncertainty still big.

Comparison $m^{\text{pole}} \leftrightarrow M^{15}$:

$$M_{\text{peak}} = 2M_{15} + \delta M_{\text{peak},15}^{\text{LO}} + \delta M_{\text{peak},15}^{\text{NLO}} + \delta M_{\text{peak},15}^{\text{NNLO}}$$

$\mu [\text{GeV}]$	$\delta M_{\text{peak},15}^{\text{total}}$	δ^{LO}	δ^{NLO}	δ^{NNLO}
15	0.17	0.16	0.02	0.00
30	0.12	0.30	-0.09	-0.08
60	0.11	0.54	-0.32	-0.1

($\alpha_s = 0.118$, $M_{15} = 175 \text{ GeV}$)

→ assume $M_{15} = 175 \pm 0.2 \text{ GeV}$,
 $\alpha_s(M_Z) = 0.118 \pm x \cdot 0.001$:

$$m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}}) = \left[\begin{array}{cccc} 175 & - 7.58 & - 0.96 & - 0.23 \\ & \text{LO} & \text{NLO} & \text{NNLO (est.)} \\ \pm 0.2 & \pm x \cdot 0.07 & & \end{array} \right] \text{ GeV}$$

δM_{15} $\delta \alpha_s$

... more detailed analysis presented

IV. Conclusions

- NNLO corrections to $\sigma_{\text{tot}}(e^+e^- \rightarrow t\bar{t})$ @ threshold are uncomfortably large :
 - 1S - peak position
 - μ - dependence
 - normalization
 - This partly due to the use of the pole mass m_t^{pole} which is IR ambiguous.
 - With suitable short distance mass definitions like m^{PS} , m^{LS} , M^{1S} the long distance sensitivity is cured and m_t can be extracted with a precision $\lesssim 200$ MeV from the total cross section.
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