Probing Anomalous Top-Couplings at Polarized Linear Collider *

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1. Introduction

The discovery of the top quark completed the fermion spectrum

required by the standard EW theory (SM) CDF, D0

Still, it is to be seen

*Collaboration with L. Brzezinski, B. Grzadkowski (Warsaw), K. Ohkuma (Kobe) and M. Szafranski (Warsaw)

- The 3rd generation is a copy of the 1st/2nd ones ?
- Any new interactions exist in top-quark couplings ?

If any new physics exists and it violates CP, we may be able to observe its effect, because

(i) CP violation in the top-quark couplings in the SM is negligible (more than 2-loop)

(ii) A lot of information on the top quark is to be transferred to the secondary leptons without getting obscured by the hadronization effects thanks to huge m_t.

Here I would like to show our work on this topic.

2. Polarized Top Production and CP Violation

$t\bar{t}$ pairs : produced via γ/Z exchange in e^+e^-

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Handedness of t and $\bar{t}$ must be the same.

$$\Downarrow$$

Helicities of $t\bar{t}$ would be $(+-)$ or $(-+)$ if m_t were much smaller than $\sqrt{s}$

However,

$$m_t \sim 175 \text{ GeV}$$

$\Longrightarrow$ Copious productions of $(++)$ and $(--)$ states.

$\sigma_{tot}(e^+e^- \to t\bar{t}) \simeq 0.6$ pb for $\sqrt{s}$ =500 GeV (and m_t =175 GeV)

$$\Downarrow$$

$$N(-+) : N(+-) : N(--) : N(++) \simeq 4.8 : 3.4 : 0.9 : 0.9 \quad \text{(in SM)}$$

What's interesting?

- $|--\rangle$ and $|++\rangle$: transformed into each other by CP

$$\Downarrow$$

$N(--) - N(++)$: a useful measure of CP violation

3. Energy Spectra of the Final Leptons

We cannot measure $N(--) - N(++)$ directly since top decays too rapidly.

Fortunately $\cdots\cdots$

The energy spectra of $\ell^\pm$ in $e^+e^- \to t\bar{t} \to \ell^+\ell^-X/\ell^\pm X$ can be a good measure of $N(--) - N(++)$. Peskin, Chang, Arens, $\cdots\cdots$

Why? That's because $\cdots\cdots$

(1) W in $t \to bW$ tends to be polarized longitudinally

since

$$\varepsilon_L^\mu \sim k_W^\mu \quad \to \quad \bar{b}\gamma_\mu(\gamma_5)t \cdot \varepsilon_L^\mu \propto \; m_t\bar{b}(\gamma_5)t$$

(2) If tbW coupling is $V - A$, $b(\bar{b})$ is left(right)-handed

since $m_b/\sqrt{s} \simeq 0$

(3) Then W^+ is emitted preferably in the same direction as $t(h = +1)$,

and in the opposite direction to $t(h = -1)$

(4) So,

$$\left\{ \begin{array}{l} \langle \ell^+ \rangle \text{ in } t(h = +1) \\ \langle \ell^- \rangle \text{ in } \bar{t}(h = +1) \end{array} \right. \text{decay is} \left\{ \begin{array}{l} \text{larger} \\ \text{smaller} \end{array} \right.$$

$$\text{than} \left\{ \begin{array}{l} \langle \ell^+ \rangle \text{ in } t(h = -1) \\ \langle \ell^- \rangle \text{ in } \bar{t}(h = -1) \end{array} \right. \text{decay}$$

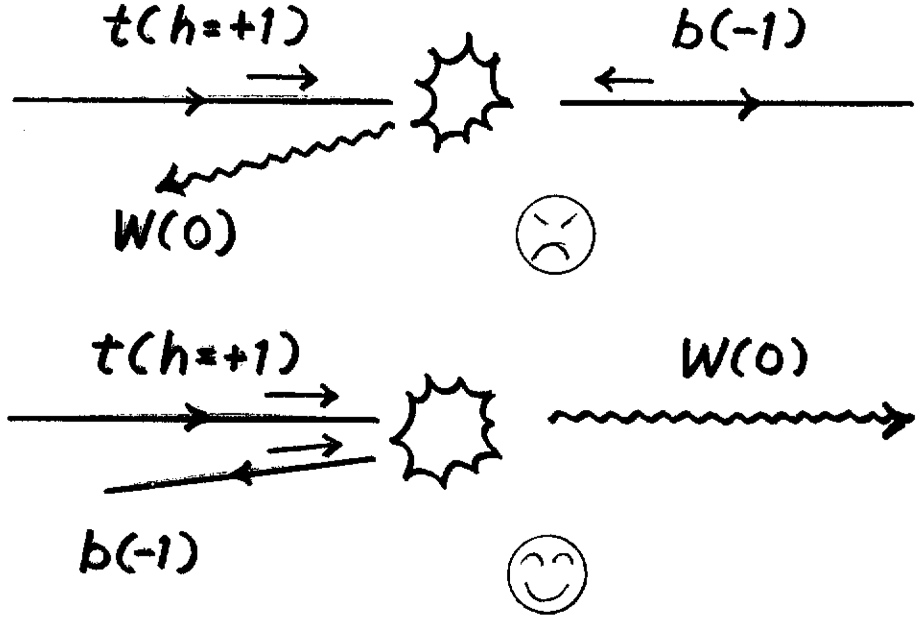
(5) Therefore

$$\left\{ \begin{array}{l} \langle \ell^+ \rangle > \langle \ell^- \rangle \\ \langle \ell^+ \rangle < \langle \ell^- \rangle \end{array} \right. \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} N(++) > N(--) \\ N(++) < N(--) \end{array} \right.$$

4. Effects of Non-standard Top Decay

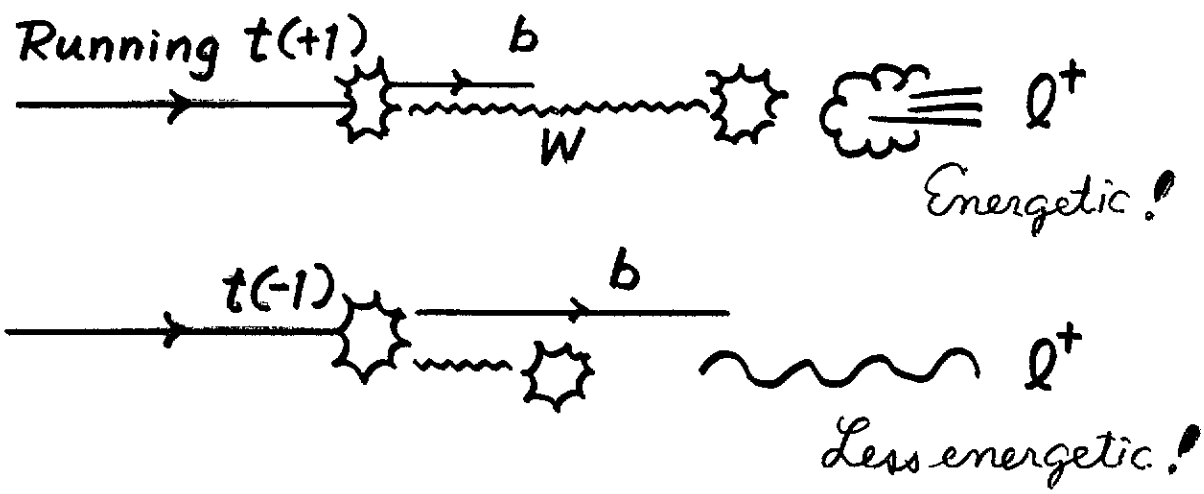
If no CP violation in the tbW vertex,

$$a(x) \equiv \frac{d\sigma^-/dx - d\sigma^+/dx}{d\sigma^-/dx + d\sigma^+/dx}$$



$$t(h=+1) \rightarrow b(h=-1)\, W(h=0)$$

$$\Downarrow$$



is known to be proportional to

$$\delta \equiv [\, N(--) - N(++)\,]/N(all).$$

Here, $x \equiv 2E\sqrt{(1-\beta)/(1+\beta)}/m_t$ with E being the energy of $\ell^\pm$ in the e^+e^- c.m. system and $\beta \equiv \sqrt{1-4m_t^2/s}$.

In this case, $a(x)$ would serve as a useful observable.

However, if CP non-conservation occurs in the $t\bar{t}\gamma/Z$ vertex, it is natural to assume that another CP non-conservation might occur in the tbW too.

So, we first computed $a(x)$ assuming that both the $t\bar{t}\gamma/Z$ vertices and the tbW vertex include non-standard CP-violating form factors:

$$\Gamma^\mu = \frac{g}{2}\bar{u}(p_t)\Big[\gamma^\mu(A_v - B_v\gamma_5) + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t}(C_v - D_v\gamma_5)\Big]v(p_{\bar{t}}),$$

$$\Gamma^\mu = -\frac{g}{\sqrt{2}}\,\bar{u}(p_b)\Big[\gamma^\mu(f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(f_2^L P_L + f_2^R P_R)\Big]u(p_t),$$
$$\bar{\Gamma}^\mu = -\frac{g}{\sqrt{2}}\,\bar{v}(p_{\bar{t}})\Big[\gamma^\mu(\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(\bar{f}_2^L P_L + \bar{f}_2^R P_R)\Big]v(p_{\bar{b}}),$$

where $v = \gamma/Z$, $P_{L/R} \equiv (1 \mp \gamma_5)/2$.

$a(x)$ becomes thereby

$$a(x) = \frac{-2(\delta/\beta)\,g(x) + \mathrm{Re}(f_2^R - \bar{f}_2^L)[\,\delta f(x) + \eta\,\delta g(x)\,]}{2\,[\,f(x) + \eta\, g(x)\,]}.$$

where $f(x)$ and $g(x)$ are those computed in previous papers, while $\delta f(x)$ and $\delta g(x)$ are what we computed.

Measuring asymmetries like $a(x)$ in this case is not easy since it is differential and therefore the expected statistics cannot be high

5. CP-violating Asymmetry

As an example of integrated quantities, we introduced

$$A_{\ell\ell} \equiv \Big(\iint_{x<\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} - \iint_{x>\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}}\Big) \Big/ \iint dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}},$$

where x and $\bar{x}$ are the reduced energies of ℓ^+ and ℓ^- respectively.

It becomes

$$A_{\ell\ell} = 0.3089\,\mathrm{Re}(f_2^R - \bar{f}_2^L) + 0.3638\,\mathrm{Re}(D_\gamma) + 0.0609\,\mathrm{Re}(D_Z)$$
$$= 0.3089\,\mathrm{Re}(f_2^R - \bar{f}_2^L) - 0.3441\,\xi,$$

where ξ is given by $\xi = -\delta/\beta$. : characterizing $\not{CP}$ in $t\bar{t}\gamma/Z$

For $\mathrm{Re}(f_2^R) = -\mathrm{Re}(\bar{f}_2^L) = \mathrm{Re}(D_\gamma) = \mathrm{Re}(D_Z) = 0.2$, e.g., we have

$$A_{\ell\ell} = 0.2085$$

and its statistical error for $N_{\ell\ell}$ events is estimated to be

$$\Delta A_{\ell\ell} = \sqrt{(1 - A_{\ell\ell}^2)/N_{\ell\ell}} = 0.9780/\sqrt{N_{\ell\ell}}.$$

$\sigma_{e\bar{e}\to t\bar{t}} = 0.60$ pb for $\sqrt{s} = 500$ GeV

$$\Downarrow$$

$$N_{\ell\ell} = 600\,\epsilon_{\ell\ell} L B_\ell^2$$

where

$\epsilon_{\ell\ell}$: $\ell^+\ell^-$ tagging efficiency

L : the integrated luminosity in fb^{-1} unit

B_ℓ ($\simeq 0.22$) : the leptonic branching ratio for t.

$$\Downarrow$$

$$|A_{\ell\ell}|/\Delta A_{\ell\ell} = 5.7 \quad \text{for } L = 50 \text{ fb}^{-1} \text{ and } \epsilon_{\ell\ell} = 0.5$$

$$\Downarrow$$

We can confirm $A_{\ell\ell}$ to be non-zero at 5.7σ level concerning the statistical uncertainty.

6. Precision of Parameter Measurements

ξ and $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ should be separately determined for more detailed studies.

How can we do it?

$\Longrightarrow$ Optimal method (by Gunion et al.)

Gunion et al., Atwood, Soni

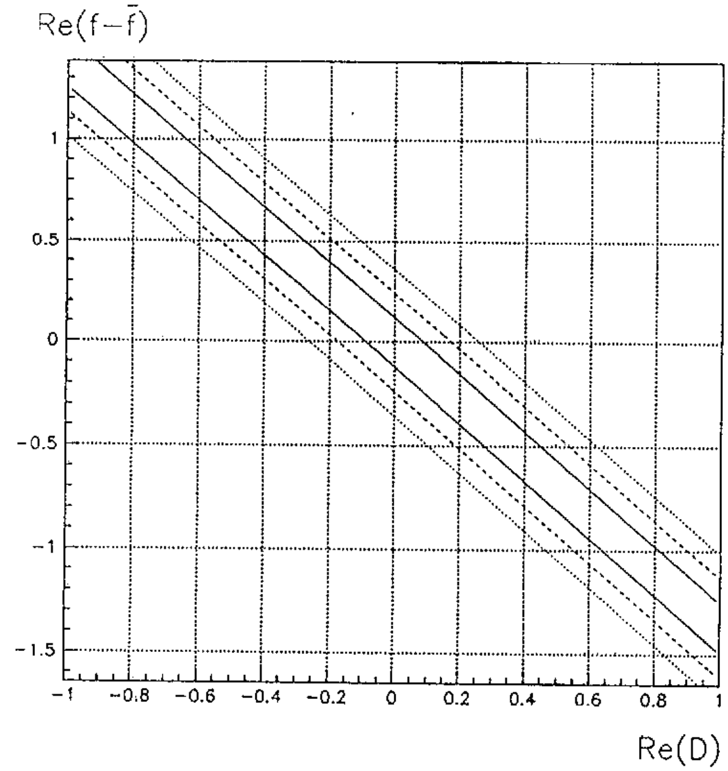


Figure 1: We can confirm the asymmetry $A_{\ell\ell}$ to be non-zero at 1σ, 2σ and 3σ level when the parameters $\mathrm{Re}(f_2^R - \bar{f}_2^L)$ and $\mathrm{Re}(D_{\gamma,Z})$ are outside the two solid lines, dashed lines and dotted lines respectively.

(We assumed $\mathrm{Re}\, D_\gamma = \mathrm{Re}\, D_Z$)

Brief summary of this method: When we have a cross section

$$\frac{d\sigma}{d\phi}(\equiv \Sigma(\phi)) = \sum_i c_i f_i(\phi)$$

where the $f_i(\phi)$ are known functions of the final-state phase space ϕ and the c_i are model-dependent coefficients.

Determining c_i

$\Longrightarrow$ Take appropriate weighting functions $w_i(\phi)$ such that

$$\int w_i(\phi)\Sigma(\phi)d\phi = c_i$$

Different choices for $w_i(\phi)$ are possible

But there is a unique choice such that the resultant statistical error is minimized.

Such functions are given by

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\Sigma(\phi)\,,$$

where X_{ij} is the inverse matrix of M_{ij} which is defined as

$$M_{ij} \equiv \int \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)} d\phi\,.$$

With them, the statistical uncertainty of c_i becomes

$$\Delta c_i = \sqrt{X_{ii}\,\sigma_T/N}\,,$$

where $\sigma_T \equiv \int (d\sigma/d\phi)d\phi$ and $N = L_{\text{eff}}\sigma_T$ is the total number of events, with L_{eff} being the integrated luminosity times efficiency

7. Use of Polarized Beam(s)

Let us express the lepton spectra as

$$\frac{1}{\sigma}\frac{d\sigma^{\pm}}{dx} = c_1^{\pm} f_1(x) + c_2^{\pm} f_2(x) + c_3^{\pm} f_3(x)$$

where

$$c_1^{\pm} = 1, \; c_2^{\pm} = \lambda \mp \xi, \; c_3^{+} = \text{Re}(f_2^{R}), \; c_3^{-} = \text{Re}(\bar{f}_2^{L})$$

and λ is another combination of $t\bar{t}\gamma/Z$ form factors.

We then apply the optimal procedure to those spectra.

$C_1^{\pm} = 1$: Standard-model contribution

$$f_1(x) = f(x) + \eta\, g(x) \quad , \quad f_2(x) = g(x)$$

$$f_3(x) = \delta f(x) + \eta\, \delta g(x)$$

λ, ξ, η : Polarization-dependent

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
$\Delta c_2^{\pm}$	0.13	0.16	0.12	0.09	0.09	0.08	0.07
$\Delta c_3^{\pm}$	0.08	0.10	0.08	0.06	0.06	0.05	0.05
N_ℓ	7676	6259	5409	4843	9093	9943	10509
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
$\Delta c_2^{\pm}$	0.13	0.11	0.08	0.07	0.07	0.05	0.05
$\Delta c_3^{\pm}$	0.08	0.07	0.05	0.04	0.05	0.04	0.03
N_ℓ	7676	6762	8055	9685	12429	17122	21019

Table 1: Expected statistical errors in $c_{2,3}^{\pm}$ measurements and the number of the single-lepton-inclusive events N_ℓ for beam polarization
(1) $P_{e^+} = 0$ vs $P_{e^-} = 0, \pm 0.5, \pm 0.8$ and ± 1,
(2) $P_{e^+} = P_{e^-} (\equiv P_e) = 0, \pm 0.5, \pm 0.8$ and ± 1
at $\sqrt{s} = 500$ GeV.
N_ℓ has been estimated within the SM for $\epsilon_\ell = 0.6$ and $L = 100$ fb^{-1}

	(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
	c_2^+	0.39	0.36	0.28	0.17	0.38	0.36	0.34
	c_2^-	0.14	0.16	0.12	0.05	0.09	0.06	0.03
	$\lvert c_2^+\rvert/\Delta c_2^\pm$	3.03	2.31	2.25	1.83	4.10	4.65	4.96
*	$\lvert c_2^-\rvert/\Delta c_2^\pm$	1.11	1.04	1.01	0.58	1.00	0.75	0.49
	(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
	c_2^+	0.39	0.28	0.19	0.17	0.36	0.35	0.34
	c_2^-	0.14	0.12	0.06	0.05	0.06	0.04	0.03
	$\lvert c_2^+\rvert/\Delta c_2^\pm$	3.03	2.52	2.47	2.59	5.20	6.30	7.01
*	$\lvert c_2^-\rvert/\Delta c_2^\pm$	1.11	1.13	0.86	0.81	0.83	0.68	0.70

Table 2: Statistical significance of $c_2^\pm$ measurement for beam polarization
(1) $P_{e^+} = 0$ vs $P_{e^-} = 0, \pm 0.5, \pm 0.8$ and ± 1, and
(2) $P_{e^+} = P_{e^-} (\equiv P_e) = 0, \pm 0.5, \pm 0.8$ and ± 1,
and the parameter set
(a) $\mathrm{Re}(A_\gamma) = \mathrm{Re}(A_Z) = \mathrm{Re}(B_\gamma) = \mathrm{Re}(B_Z) = \mathrm{Re}(C_\gamma) = \mathrm{Re}(C_Z) = \mathrm{Re}(D_\gamma) = \mathrm{Re}(D_Z) = 0.1$ at $\sqrt{s} = 500$ GeV

* Precision of c_2^- measurement in this case gets worse for $P_e \neq 0$

(1) P_{e^-}	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2^+	0.17	0.31	0.46	0.61	0.11	0.08	0.07
c_2^-	$-4\cdot10^{-3}$	0.04	0.11	0.19	−0.01	10^{-3}	0.01
$\lvert c_2^+\rvert/\Delta c_2^\pm$	1.33	1.97	3.70	6.63	1.15	1.07	1.02
$\lvert c_2^-\rvert/\Delta c_2^\pm$	0.03	0.24	0.86	2.09	0.06	0.02	0.10
(2) P_e	0	+0.5	+0.8	+1.0	−0.5	−0.8	−1.0
c_2^+	0.17	0.46	0.59	0.61	0.08	0.07	0.07
c_2^-	$-4\cdot10^{-3}$	0.11	0.18	0.19	10^{-3}	0.01	0.01
$\lvert c_2^+\rvert/\Delta c_2^\pm$	1.33	4.14	7.86	9.38	1.20	1.31	1.44
$\lvert c_2^-\rvert/\Delta c_2^\pm$	0.03	0.97	2.40	2.95	0.02	0.12	0.14

Table 3: Statistical significance of $c_2^\pm$ measurement for beam polarization
(1) $P_{e^+} = 0$ vs $P_{e^-} = 0, \pm0.5, \pm0.8$ and ±1, and
(2) $P_{e^+} = P_{e^-} (\equiv P_e) = 0, \pm0.5, \pm0.8$ and ±1,
and the parameter set
(b) $\mathrm{Re}(A_\gamma) = -\mathrm{Re}(A_Z) = \mathrm{Re}(B_\gamma) = -\mathrm{Re}(B_Z) = \mathrm{Re}(C_\gamma) = -\mathrm{Re}(C_Z) = \mathrm{Re}(D_\gamma) = -\mathrm{Re}(D_Z) = 0.1$ at $\sqrt{s} = 500$ GeV.

8. Summary

We discussed the non-standard CP-violating interactions in the $t\bar{t}$ productions and their subsequent decays.

- We have assumed the most general CP-violating interactions both in the production and in the decay vertices in order to perform a consistent analysis.
- We calculated the final-lepton-energy spectra

$$\frac{d\sigma}{dx d\bar{x}}, \quad \frac{d\sigma^{\pm}}{dx}$$

- We applied the optimal procedure to estimate the expected precision of anomalous couplings.
- We found the use of polarized $e\bar{e}$ beam(s) could improve the precision, but the details depend on models we are testing.

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