NLO corrections to $B$ decays in $k_T$ factorization

Hsiang-nan Li
Institute of Physics, Academia Sinica, Taiwan

Feb. 17, 2004
APPI 2004

- Introduction
- Collinear vs. $k_T$ Factorizations
- Example Study: $B$ Meson Wave Function
- Conclusion
**Introduction**

- Exclusive $B$ meson decays are important for measuring standard-model parameters and for exploring new physics.

- The currently available theories include perturbative QCD (PQCD), OCD-improved factorization (QCDF), soft-collinear effective theory (SCET), and light-cone sum rules (LCSR). For a review, see Li 03.

- PQCD is based on $k_T$ factorization theorem (Lu’s talk), while the others on collinear factorization theorem.

- Experimental precision improves, and theoretical precision should too.

- NLO calculation is necessary for checking the convergence of perturbation theories, and for reducing the scale dependence of LO predictions.

- Some discrepancies have been observed. Need to understand whether they can be resolved. NLO calculation is necessary for confirming new physics (Cheon and Keum’s talks).

- I will compare the frameworks: collinear vs. $k_T$ factorization, and present an example study—the $O(\alpha_s)$ corrections to the $B$ meson wave function. This wave function is the key input of the theories of exclusive $B$ decays.

- The conclusions on the asymptotic behavior of the $B$ meson wave function from the two frameworks are completely different.
Collinear vs. $k_T$ Factorizations

- Factorization theorems
  - two types of factorization theorems for exclusive processes in PQCD: collinear factorization and $k_T$ factorization.
  - At large momentum transfer $Q$, an exclusive process can be calculated as an expansion of $\alpha_s(Q)$ and $\Lambda/Q$, $\Lambda$: a small scale.
  - pion form factor in $\pi(P_1) + \gamma^*(q) \rightarrow \pi(P_2)$,

$$P_1 = (P_1^+, 0, 0_T), \quad P_2 = (0, P_2^-, 0_T), \quad Q^2 = 2P_1^+ P_2^-$$

$$F_{\pi} = \phi_\pi \otimes H^{(0)} \otimes \phi_\pi + \phi_\pi \otimes H^{(1)} \otimes \phi_\pi + \phi_\pi \otimes H^{(0)} \otimes \phi_\pi$$

$\phi_\pi$: nonperturbative pion distribution amplitude
$H^{(0),(1)}$: perturbative hard kernels
$\phi_p$: twist-3 pion distribution amplitude
Collinear factorization

- pion form factor as an example for deriving theorem
- at $O(\alpha_s)$, $H^{(0)}(x_1, x_2) \propto 1/(x_1 P_1 - x_2 P_2)^2 = -1/(x_1 x_2 Q^2)$

At $O(\alpha_s^2)$, collinear divergence is generated in a loop integral, and needs to be factorized into the pion distribution amplitude.

- $l \parallel P_1 \Rightarrow l^+ \sim P_1^+ \gg l_T \sim \Lambda \gg l^- \sim \Lambda^2/P_1^+, P_1^2 \sim l^2 \sim O(\Lambda^2)$. 
  $\Rightarrow$ an on-shell (infrared) gluon.
\[ H^{(0)} \propto \frac{1}{(x_1 P_1 - x_2 P_2 + l)^2} \]

\[ = \frac{1}{(x_1 P_1 - x_2 P_2)^2 + 2x_1 P_1^+ l^- - 2x_2 P_2^- l^+ + 2l^+ l^- - l_T^2} . \]

dropping \( l^- \) and \( l_T \)

\[ H^{(0)}(\xi_1, x_2) \propto \frac{1}{2x_1 x_2 P_1^+ P_2^- + 2x_2 P_2^- l^+} \]

\[ = \frac{1}{2(x_1 + l^+/P_1^+)x_2 P_1^+ P_2^-} \equiv \frac{1}{\xi_1 x_2 Q^2} . \]

• factorization theorem \( \Rightarrow \) convolution only in the longitudinal component of parton momentum,

\[ F_\pi^{(1)} = \int d\xi_1 d\xi_2 \phi_\pi^{(1)}(\xi_1) H^{(0)}(\xi_1, x_2) \phi_\pi^{(0)}(x_2) . \]

\( \phi_\pi^{(1)} \) contains integration over \( l^-, l_T \).

• factorization to all orders \( \Rightarrow \) collinear factorization

\[ F_\pi = \int d\xi_1 d\xi_2 \phi_\pi(\xi_1) H(\xi_1, \xi_2) \phi_\pi(\xi_2) . \]

• SCET, LCSR, and QCDF are based on collinear factorization.
**$k_T$ Factorization**

- In the small $x$ region, $x_1 x_2 Q^2$ is small. Dropping only $l^-$ (keeping $l_T$) in $H^{(0)}$

\[
H^{(0)}(\xi_1, x_2, l_T) \propto \frac{1}{2(x_1 + l^+/P_1^+) x_2 P_1^+ P_2^- + l_T^2} \equiv \frac{1}{\xi_1 x_2 Q^2 + l_T^2}
\]

- Factorization theorem $\Rightarrow$ convolution in both the longitudinal and transverse components of parton momentum

\[
F^{(1)}_\pi = \int d\xi_1 dx_2 d^2l_T \phi^{(1)}_\pi(\xi_1, l_T) H^{(0)}(\xi_1, x_2, l_T) \phi^{(0)}_\pi(x_2).
\]

- Factorization to all orders $\Rightarrow k_T$ factorization,

\[
F_\pi = \int d\xi_1 d\xi_2 d^2k_1T d^2k_2T \phi_\pi(\xi_1, k_1T) H(\xi_1, \xi_2, k_1T, k_2T) \phi_\pi(\xi_2, k_2T).
\]

- Collinear factorization is suitable, if the small $x$ region is not important.
- $k_T$ factorization is more general, suitable also in the small $x$ region.
- Have shown that $k_T$ factorization is gauge invariant (Nagashima and Li 03)
- PQCD is based on $k_T$ factorization.
**Example Study: B Meson Wave Function**

- **B meson distribution amplitude**
  - Collinear factorization can be derived for exclusive $B$ meson decays
    ⇒ $B$ meson distribution amplitude with $y = (0, y^-, 0_\perp),$  
    \[
    \langle 0 | \bar{q}(y) W_y(n_-) \Gamma h_0) | \bar{B}(P) \rangle 
    = -\frac{f_B \sqrt{m_B}}{2} \phi_+(v \cdot y, \mu) \text{tr} \left( n_- \Gamma \frac{1+ \not{\delta}}{2} \gamma_5 \right),
    \]
  
  with the Wilson line operator,
  
  \[
  W_y(n_-) = P \exp \left[ -ig \int_0^\infty d\lambda n_- \cdot A(y + \lambda n_-) \right].
  \]

- The lowest-order evolution kernel for the $B$ meson distribution amplitude $\delta(k^+ - k'^+).$ Show some $O(\alpha_s)$ diagrams (a)-(d) and the all-order diagrams (e).
  
  \[
  Z_{ab}^{(1)}(k^+, k'^+, \mu) = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{n_- \cdot v}{v \cdot l^2 n_- \cdot l} \times \left[ \delta(k^+ - k'^+ + l^+) - \delta(k^+ - k'^+) \right].
  \]

$k^+$ being the light spectator momentum.
NLO corrections to $B$ decays in $k_T$ factorization (page 8)

Hsiang-nan Li
Institute of Physics, Academia Sinica, Taiwan
• The loop integral leads to the counterterm (Lange, Neubert 03),

\[- \frac{\alpha_s C_F}{2\pi} \frac{1}{\epsilon} \left[ \frac{k^+ \theta(k^{'+} - k^+)}{k^{'+} (k^{'+} - k^+)_+} + \frac{\theta(k^+ - k^{'+})}{(k^+ - k^{'+})_+} \right],\]

where $1/\epsilon$ comes from the integration over $l_T$.  

• The corresponding anomalous dimension, the splitting kernel, determines the asymptotic behavior,

\[\phi_+(k^+, \mu) \sim 1/k^+, \text{ for } k^+ \to \infty.\]

• The evolution effect ruins the normalizability of the $B$ meson distribution amplitude

$\Rightarrow f_B$ is not well-defined.
B meson wave function

- Collinear factorization for the semileptonic decay \( B(P_1) \to \pi(P_2)l\nu(q) \),

\[
F_{B\pi} \propto \int dx_1 dx_2 \phi_B(x_1) \frac{1 + 2x_2}{x_1 x_2^2} \phi_\pi(x_2)
\]

\[\Rightarrow \text{log divergence for } \phi_\pi \propto x(1 - x)\]

- end-point singularity in collinear factorization
  \[\Rightarrow \text{breakdown of collinear factorization}\]
  \[\Rightarrow k_T \text{ factorization for the end-point (small } x \text{) region.}\]
  \[\Rightarrow B \text{ meson wave function with } y = (0, y^-, b),\]

\[
\langle 0 | \bar{q}(y) W_y(n_-) \gamma^\dagger W_0(n_-) \not{\! h} - \Gamma h(0) | \bar{B}(P) \rangle = -i f_B \sqrt{m_B} \tilde{\phi}_+(v \cdot y, b, \mu) \text{tr} \left( \not{\! h} - \Gamma \frac{1 + \not{\! b}}{2} \gamma_5 \right)
\]
In $k_T$ factorization, the modified Feynman rule for Figs. (a) and (b) is

$$
Z_{ab}^{(1)}(k^+, k'^+, b, \mu) = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{n_\perp \cdot v}{v \cdot l l^2 n_\perp \cdot l} \times \left[ \delta(k^+ - k'^+ + l^+) \exp(-i l_T \cdot b) - \delta(k^+ - k'^+) \right].
$$

Note the extra Fourier factor $\exp(-i l_T \cdot b)$ in the conjugate $b$ space, which suppresses the large $l_T$ region ⇒ the integral is UV finite.

$$
- \frac{\alpha_s C_F}{\pi} \left[ \frac{\theta(k'^+ - k^+)}{(k'^+ - k^+)_+} K_0((k'^+ - k^+)b) - \frac{\theta(k^+ - k'^+)}{(k^+ - k'^+)_+} K_0((k^+ - k'^+)b) \right].
$$

The evolution of the $B$ meson wave function does not involve a splitting kernel, and does not change the $k^+$ distribution,

$$
\phi_+(k^+, b, \mu) = R(\mu, b)\phi_+(k^+, b).
$$

$R(\mu, b)$ being the RG factor.

The $B$ meson distribution amplitude is the small $b$ limit, $b \to 1/m_B \to 0$, at which the Bessel function $K_0$ remains UV finite.
Conclusion

• NLO calculation in $k_T$ factorization is a bit more complicated than in collinear factorization (due to the Fourier factor), but do-able.

• NLO corrections in $k_T$ factorization are gauge-invariant (not discussed here). See Collins 03.

• NLO corrections to the $B$ meson wave function produce the leading (double) and next-to-leading (single) Sudakov logarithms (not discussed here).

• The evolution of the $B$ meson wave function in $k_T$ factorization is well-behaved. It does not ruin the normalizability of the $B$ meson wave function.