First Simulation of IP-BPM with MAFIA

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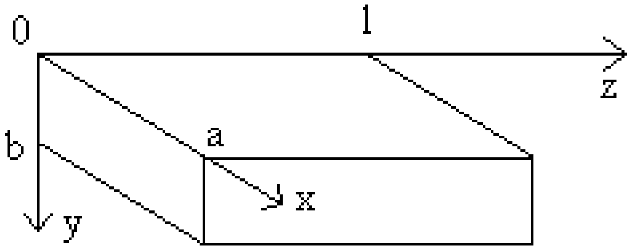
Contents

1 Introduction

Using the electromagnetic simulation program MAFIA, we did a study of the IP-BPM (Interation Point - Beam Position Monitor). The purpose of this BPM is to measure an electron beam's position with a precision of e few nanometers (nm). This is a particular BPM, since it is made of a rectagular cavity. For more information about how rectangular cavity BPM work, please refer to [1].

2 Theoretical calculations

Here, we will use a simple rectangular cavity to do some basic calculations of what signal we expect of an electron beam passing through the cavity.



The values for IP-BPM are : a = 61.4 mm, b = 48.56 mm and l = 6 mm±0.05 mm. For a given mode N=(m,n,q) where we assume q=0, the following electric field in the cavity exists :

$$\overrightarrow{E} = E_0 sin(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) \exp(i\omega t)\overrightarrow{u_z} \quad (1)$$

The most important parameter in the cavity is the normalized impedance $\left(\frac{R_s}{Q_0}\right)_N$ since it only depends on the cavity parameters. Moreover, for TM_{100} and TM_{010} modes (the two dipoles modes), this value is proportional to the beam offset and the beam inclination as prooved in [1]. We are going to write this factor so that it will be obvious that it only depends on cavity parameters, using the following formula for the N^{th} mode :

$$\left(\frac{R_s}{Q_0}\right)_N = \frac{V_N^2}{\omega_N W_N} \quad (2)$$

where ω_N is the pulsation of the N^{th} mode, given by :

$$\omega_N = \pi c \sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \quad (3)$$

V_N is the voltage induced by the electric field, given by :

$$V_N = E_0 sinc(\frac{kl}{2}) l \sin(\frac{m\pi x}{a}) \sin(\frac{n\pi y}{b}) \qquad (4)$$

and W_n is the energy of the electric field. With equation 1, we can calculate W_N :

$$W_N = 2 \times \frac{\epsilon}{2} \int_{cavity} \Re(\overrightarrow{E})^2 d\tau \qquad (5)$$

We can separate the variables x, y and z to get the simple form :

$$W_N = \epsilon\ E_0^2 \int_0^a \sin(\frac{m\pi x}{a})^2 dx \int_0^b \sin(\frac{n\pi y}{b})^2 dy \int_{-l/2}^{l/2} \cos(kz)^2 dz \qquad (6)$$

So we obtain the following expression for the energy stored in the cavity for a given mode N :

$$W_N = \epsilon\ E_0^2\ \frac{abl}{8}\ (1 + sinc(\frac{kl}{2}) \cos(\frac{kl}{2})) \qquad (7)$$

Combining 3, 4 and 7, we can see that the normalized impedance only depends on the parameters of the cavity.

$$\left(\frac{R_s}{Q_0}\right)_N = \frac{8\ sinc(\frac{kl}{2})^2\ l}{1 + sinc(\frac{kl}{2}) \cos(\frac{kl}{2})}\ \frac{\sin(\frac{m\pi x}{a})^2 \sin(\frac{n\pi y}{b})^2}{\omega_N\ \epsilon\ a\ b} \qquad (8)$$

Here are the values for the first TM modes of the IP-BPM :

TM mode	Frequency	Offset	R/Q (Ω)	Vout
TM_{000} (1^{st} common mode)	3.938 GHz	0	36.74	23.15 V (40.3 dBm)
TM_{100} (X dipole mode)	5.781 GHz	1 nm	$2.62e^{-13}$	2.67 μV (-98.5 dBm)
		1 μm	$2.62e^{-7}$	2.67 mV (-38.5 dBm)
		1 mm	0.261	2.66 V (21.5 dBm)
		10 mm	18.24	22.6 V (40.1 dBm)
TM_{010} (Y dipole mode)	6.643 GHz	1nm	$3.64e^{-13}$	3.35 μV (-96.5 dBm)
		1 μm	$3.64e^{-7}$	3.35 mV (-36.5 dBm)
		1 mm	0.362	3.34 V (23.5 dBm)
		10 mm	20.24	25.0 V (41.0 dBm)

Where V_{out} is the output voltage of the cavity, given by the formula :

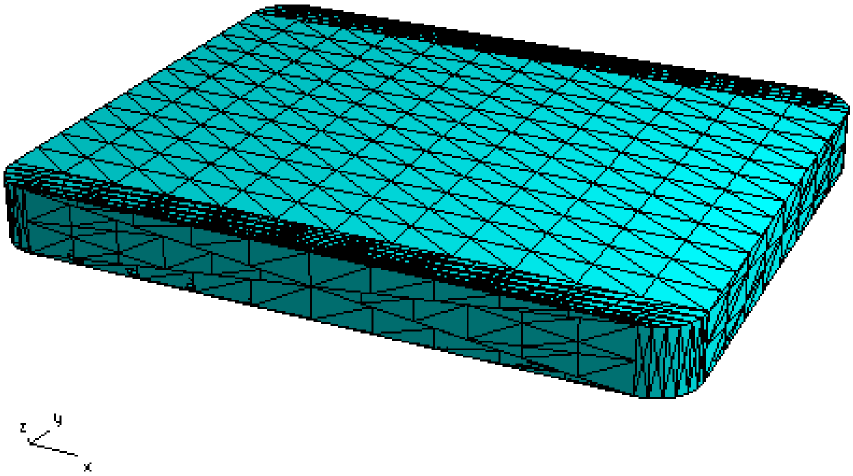
$$V_{N,out} = \frac{q\omega_N}{2} \sqrt{\frac{Z}{Q_{N,ext}} \left(\frac{R_s}{Q_0}\right)_N} \qquad (9)$$

Now we are going to compare these results with those given by the electromagnetic simulation software MAFIA. However these first results show that the measurements must be very precise to achieve a nanometric resolution.

3 Estimation of Q_0, Q_L and Q_{ext} values with MAFIA

3.1 First model of the cavity alone

First of all, we have to check if the first TM modes have the same frequency than what we theoretically calculated. Indeed, due to fabrication impossibilities, the cavity is slightly different from a real rectangular design as seen on the MAFIA screen below.



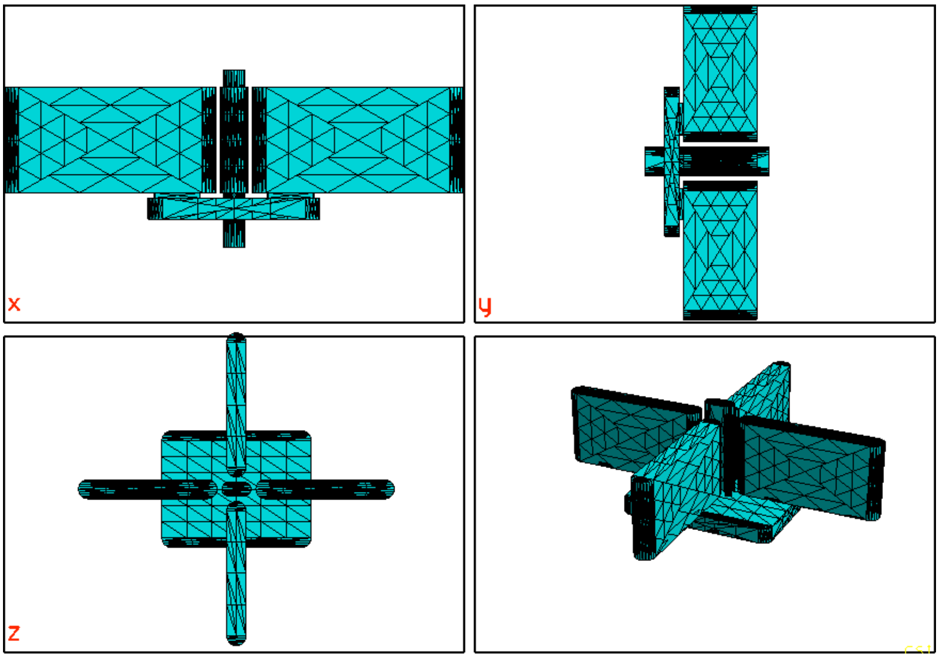
This model gives the following mode frequencies and quality factors :

Mode	Frequency	Comment	Q_0
1	3.936 GHz	1st common mode	37697
2	5.777 GHz	TM_{10} mode	39512
3	6.639 GHz	TM_{01} mode	55589
4	7.872 GHz	2nd common mode	53428
5	7.948 GHz	2nd common mode	43945

As this table shows, there is an unexpected mode (the 5th) which may have appeared because of the little geometry changes in the cavity. But we may say that the 4th and 5th modes found are the same mode (2nd common mode of the cavity). Moreover, the quality factor is maybe too high because the model doesn't take into account the holes for a beampipe that will be included inside the cavity.

3.2 Cavity with the external coupling circuit

To estimate the loaded quality factor Q_L, we need to draw the entire cavity with the coupling circuit. This external circuit consists of 4 slots that couple the electromagnetic field of the cavity with waveguides. In these waveguides, there will be an antenna that will measure the electric field to give the output voltage V_{out}.



Since the calculations of this structure gives Q_L for the first modes, we can deduce the Q_{ext} value of the coupling circuit using the previous values of Q_0. Note that there are more modes than before due to the addition of slots, beampipe and waveguide. But the TM_{00}, TM_{10} and TM_{01} can still be seen.

Mode	Frequency	Comment	Q_L
1	3.958 GHz	Cavity, 1st common mode	4704
2	5.608 GHz	Y-Waveguide, 1st common mode	6119
3	5.609 GHz	Y-Waveguide, 1st common mode	6134
4	5.6686 GHz	X-Waveguide, 1st common mode	6129
5	5.6688 GHz	X-Waveguide, 1st common mode	6131
6	5.720 GHz	Cavity, TM_{100} mode	5373
7	6.443 GHz	Cavity, TM_{010} mode	6071
8	7.152 GHz	Y-Waveguide, TM_{010} mode	7288
9	7.154 GHz	Y-Waveguide, TM_{010} mode	7308
10	7.327 GHz	X-Waveguide, TM_{100} mode	7213
11	7.333 GHz	X-Waveguide, TM_{100} mode	7273
12	7.714 GHz	Cavity, 2nd common mode	6339
13	7.941 GHz	Cavity, 2nd common mode	6855

Now, it is possible to calculate the external quality factors of the first modes using :

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \tag{10}$$

This gives us the following values :

Mode	Frequency	Q_L	Q_0	Q_{ext}
TM_{00}	3.958 GHz	749	6000	5372
TM_{10}	5.720 GHz	855	6289	6214
TM_{01}	6.443 GHz	966	8847	6811
$TM_{11}/1$	7.714 GHz	1009	8503	7188
$TM_{11}/2$	7.941 GHz	1091	6994	8118

4 Response of the cavity to a beam offset

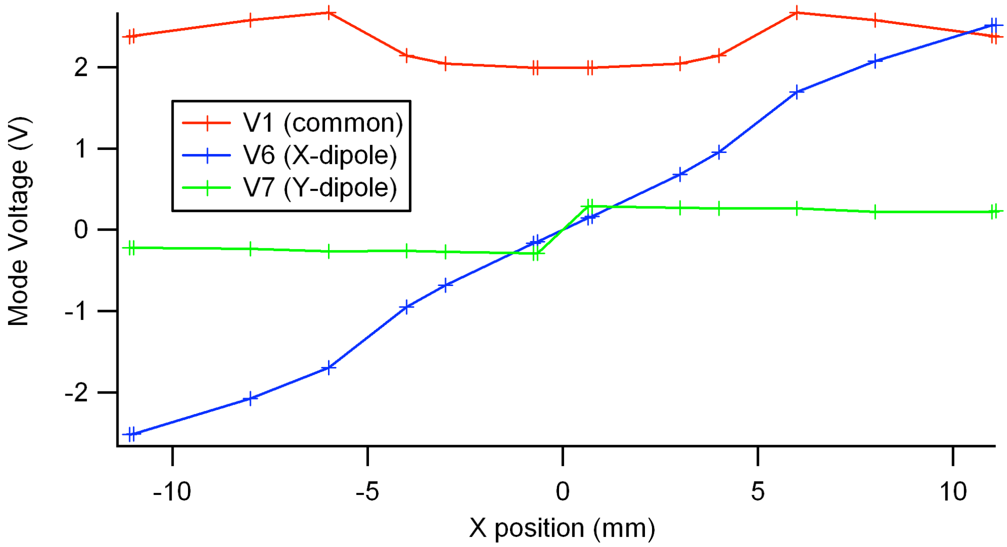
Now, we are going to see the response of the cavity to a beam offset. Note that it is difficult and quite impossible to achieve a nanometric resolution. Indeed, MAFIA creates automatically the meshes for the given geometry and there is a limited mesh step. Thus, at a certain point, increasing the mesh numbers and decreasing the mesh step doesn't change at all the model.

Here are the best mesh position achieved for the cavity model :

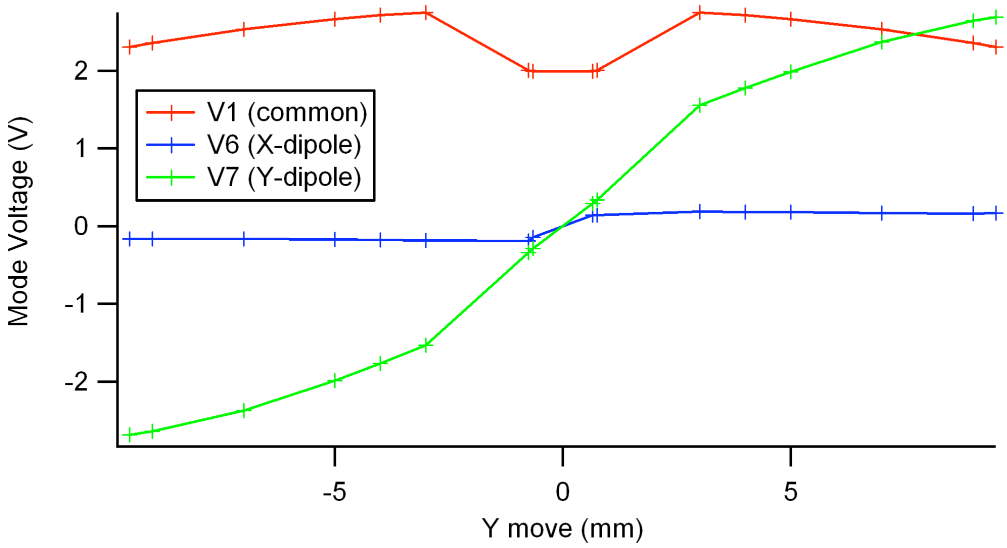
X position	Y position
0.65 mm	0.65 mm
0.75 mm	0.75 mm
3.0 mm	3.0 mm
4.0 mm	4.0 mm
6.0 mm	5.0 mm
8.0 mm	7.0 mm
11.0 mm	9.0 mm
11.1 mm	9.5 mm

MAFIA gives us the possibility to calculate the voltage of a given mode along a tajectory (definded by meshes). So we chose to calculate along the cavity on the different meshes described above.
Note : when X position is changing, Y position is 0.65 mm and when Y position is changing, X position is 0.65 mm, since it is not possible to have a better resolution.

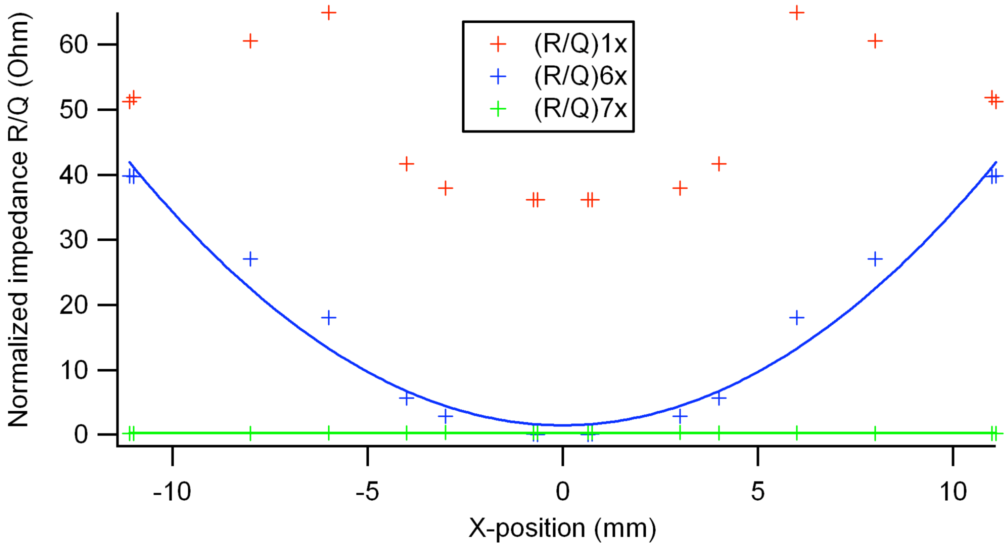
This first graph shows the mode voltage versus X position.



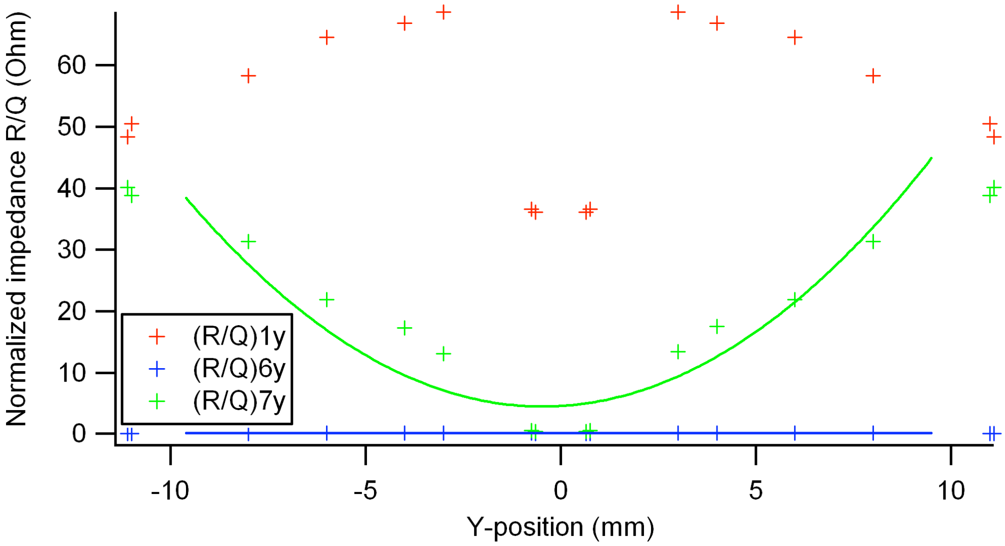
This second graph shows the mode voltage versus Y position.



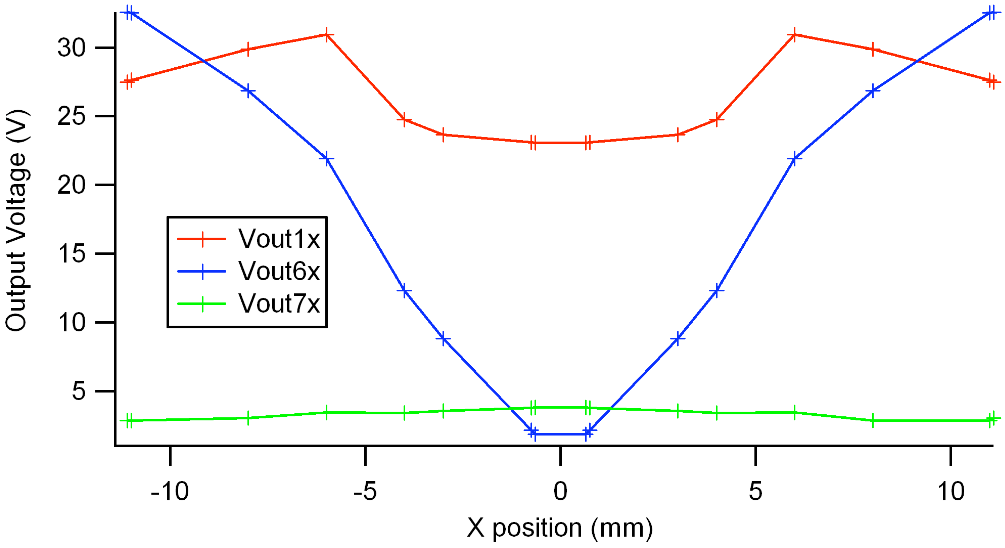
These are the brute values for the 3 principal modes. We can first see that the dipole modes are quite linear around the center of the cavity in their respective moves, which is the property that we want to use for beam monitoring and they are constant otherwise.
Now, it is possible to calculate the normalized impedances values for each positions using equation 2.
The following graph show the normalized impedances versus X position.



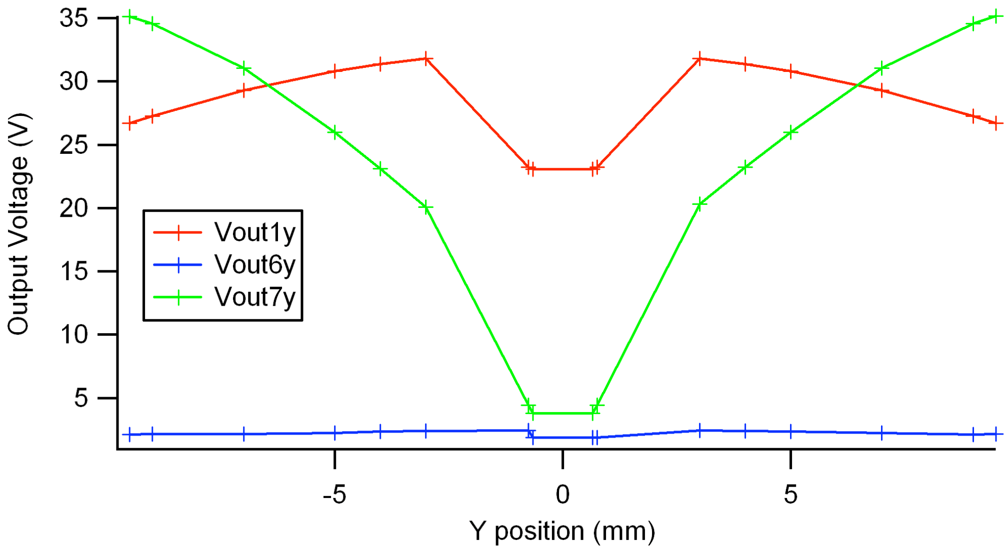
This graph shows the normalized impedances versus Y position.



And then we may calculate the output voltage with equation 9.
The following graphes show the output voltage versus X and Y position.



We can see that the output voltage is still quite linear with the beam offset.



It is good to see that the dipole mode voltage is still linear versus beam offset and that the monopole mode is quite constant along this part of the cavity. Moreover, the obtained values of V_{out} are very close to those calculated in the first section.

5 Conclusion

Due to MAFIA internal problems, it hasn't been possible to clearly check the cavity's response around a few nanometers' offset but these first results are very encouraging since the output voltage is linear around a few millimeters of the center of the cavity.

Aknowledgment

Thank you again Tauchi-san and Honda-san for all the help you provide me.

References

[1] P. Doublet, "An Introduction to rectangular Cavity Beam Position Monitors", http://acfahep.kek.jp/subg/ir/nanoBPM/index.html